ANALYTICAL ESTIMATION OF UNCERTAINTY OF COORDINATE MEASUREMENTS OF GEOMETRIC DEVIATIONS. MODELS BASED ON DISTANCE BETWEEN POINT AND STRAIGHT LINE

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S u m m a r y

The paper is the first of a series on possibility of analytical evaluation of uncertainty of coordinate measurements. The paper presents models for evaluation of uncertainty of measurement of some geometrical deviations (straightness, coaxiality and perpendicularity of an axis to a plane) basing on the formula for distance between point and straight line. An essential element of the proposed methodology is the use of minimal mathematical number of characteristic points of the measured workpiece and expressing the deviation as a function of coordinates' difference of the points.

Keywords: coordinate measuring technique, measurement uncertainty

Analityczne wyznaczanie niepewności współrzędnościowych pomiarów odchyłek geometrycznych. Modele bazujące na odległości punktu od prostej

S t r e s z c z e n i e

W artykule przedstawiono ocenę możliwości analitycznego wyznaczania niepewności pomiarów współrzędnościowych. Przyjęto modele wyznaczania niepewności niektórych odchyłek geometrycznych (prostoliniowości, współosiowości i prostopadłości osi do płaszczyzny), których podstawą są równania na odległość punktu od prostej. Istotnym elementem prezentowanej metodyki jest wykorzystanie minimalnej liczby charakterystycznych punktów mierzonego przedmiotu oraz wyrażenie odchyłki jako funkcji różnic współrzędnych tych punktów.

Słowa kluczowe: współrzędnościowa technika pomiarowa, niepewność pomiaru

1. Introduction

The most important components of measurement uncertainty which source of is the measuring machine are kinematic and probing head errors [1]. Another important component arising from the environmental conditions is temperature error caused by the temperature deviation from 20ºC as well as the spatial and time temperature gradients [1]. A simplification in the definition of geometrical

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tolerance is also an important uncertainty component coming from the operator [2].

One of the best worked out procedures of estimation of coordinate measurements' uncertainty are the experimental method described in [3] and the method using the computer simulation [4]. The algorithms used in the simulation technique are not provided to the user of the software. The document [4] requires only that the software provider describes as precisely as possible the components of the uncertainty which are included in the software, and how to properly gather and input the necessary data into the software. The developer of the most widely known simulation software is the PTB in Braunschweig. The latest version of the VCMM software takes into account kinematic errors of the machine, probing head errors, temperature effects and in a certain range the form deviations of the workpiece [5, 6].

The author of this publication developed a software for analytical evaluation of uncertainty of coordinate measurements. The current version of the software takes into consideration the influence of kinematic errors of CMM, probing head errors and temperature influences. The applied methodology uses the same mathematic model of the "measurement" of a single point and the same data as any simulation software. An essential difference is that the model of evaluation of particular features (dimensions and geometrical deviations) is based on the minimal number of points required for the definition of these features. The points are the probing points and/or the definition points of the associated elements (symmetry planes, axes, centre points). Second significant difference, which enables the use of the analytical technique is that the particular features in the applied model are functions of **coordinates' differences** of characteristic points. Next, the uncertainty of measurement of coordinates' differences can be expressed as a function of kinematic errors (including temperature effect) and probing head errors. More details on the methodology can be found in [7-10].

Closer analysis of the used methodology indicates that the model of evaluation of particular characteristics can be divided into 3 groups using 3 different formulas: distance of point and straight line, distance of point and plane and distance of two points. This publication presents formulas appropriate for tasks involving determination of distance of point from straight line.

2. The general model of distance between point and straight line

For analytical evaluation of uncertainty of indirect measurement $u_c(y)$ it is necessary to know the measurement model, i.e. a function expressing the measurand y by means of measurands x_i, $i = 1 \ldots N$, which are used to calculate the value of y and uncertainties $u(x_i)$ corresponding to measurands x_i:

$$y = f\left(x_1, x_2, \ldots, x_N\right) \tag{1}$$

$$u_c\left(y\right) = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2\left(x_i\right)} \tag{2}$$

In the presented model the geometrical deviation is expressed as a function of coordinates' differences of characteristic points of the workpiece, and the uncertainty of the coordinates' differences measurement is derived from the model describing (for the particular machine design) the error of "point measurement" in the function of kinematic errors and errors of the probing head of the measuring machine. The following form of the formula for error of "point measurement" [8] is to be used for the presented purpose:

$$\begin{bmatrix} ex \\ ey \\ ez \end{bmatrix} = \begin{bmatrix} xpx \\ xty \\ xtz \end{bmatrix} + \begin{bmatrix} ytx \\ ypy \\ ytz \end{bmatrix} + \begin{bmatrix} ztx \\ zty \\ zpz \end{bmatrix} + \begin{bmatrix} -\left(h-z\right)xry \\ \left(h-z\right)xrx \\ \left(yt-m\right)xrx \end{bmatrix} + \begin{bmatrix} -\left(yt-m\right)xrz \\ xt \cdot xrz \\ -xt \cdot xry \end{bmatrix} +$$
$$+ \begin{bmatrix} -z \cdot yry \\ -z \cdot yrx \\ \left(yt-m\right)yrx \end{bmatrix} + \begin{bmatrix} -\left(yt-m\right)yrz \\ x \cdot yrz \\ -x \cdot yry \end{bmatrix} + \begin{bmatrix} -\left(h-z\right)zry \\ \left(h-z\right)zrx \\ yt \cdot zrx \end{bmatrix} + \begin{bmatrix} -yt \cdot zrz \\ xt \cdot zrz \\ -xt \cdot zry \end{bmatrix} +$$
$$+ \begin{bmatrix} -\left(yt-m\right)xwy \\ x \cdot xwy \\ -xt \cdot xwy \end{bmatrix} + \begin{bmatrix} -\left(h-z\right)xwz \\ \left(h-z\right)ywz \\ \left(yt-m\right)ywz \end{bmatrix} + \begin{bmatrix} Px \\ Py \\ Pz \end{bmatrix} \tag{3}$$

Next, we have to find an estimation of the maximum error in measurement of coordinates' differences for each pair of points *A* and *B*, according to the B method of uncertainty evaluation:

$$\begin{bmatrix} e_{(xB-xA)} \\ e_{(yB-yA)} \\ e_{(zB-zA)} \end{bmatrix} = \begin{bmatrix} ex\left(B\right) - ex\left(A\right) \\ ey\left(B\right) - ey\left(A\right) \\ ez\left(B\right) - ez\left(A\right) \end{bmatrix} \tag{4}$$

In coordinate metrology, the distance of point *S* from straight line *p* given by a point lying on the line and a unit vector *v* parallel to the line is calculated as:

$$d(S,p)=\left|(P-S)\times v\right| \tag{5}$$

Tasks described in the following differ in number of minimal number of points required for calculation and the manner of defining the unit vector *v*. It needs to be pointed out that in all formulas the deviation is a function of coordinates' difference of points used for evaluation of the deviation.

In the following all formulas incorporate a shortened notation for the difference of coordinates of two points – e.g.: $x_{BA}=(x_B-x_A)$.

3. Models of the measurement of straightness and coaxiality deviations

The straightness of a line (axis) can be determined on the basis of the measurements of distance of a point from the straight line determined by two end points of the line segment (Fig. 1). The model refers to a simplified method of classical measurement of straightness. The minimal number of points required to determine the deviation is 3. Point *S* is to lay on the line connecting points *A* and *B* and is close to the middle of the line segment *AB*.

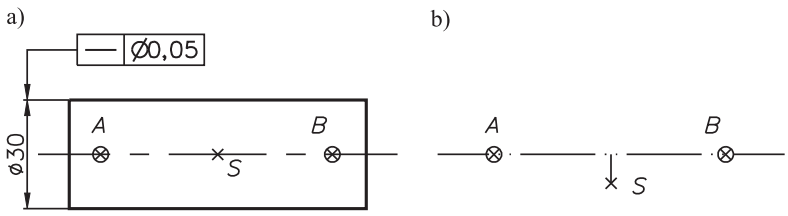

Fig. 1. Measurement of straightness deviation: a) a design drawing with characteristic points, b) measurement model

The coaxiality deviation of two straight lines (axes) can be determined as a distance *l* of a point *S* of the toleranced axis from the datum axis. The minimal number of points required to determine the deviation is 3 (Fig. 2). Here, the point *S* is to lay on the straight line connecting points *A* and *B*, usually in a significant distance from the end-points of the line segment *AB*.

The coaxiality deviation of an axis of a hole from the common axis of two holes can be determined as a distance *l* of a point *S* of the toleranced axis from the datum axis. The minimal number of points required to determine the deviation is 3 (Fig. 3). Here, the point *S* is to lay on the straight line connecting points *A* and *B*, close to one of the end-points of line segment *AB*.

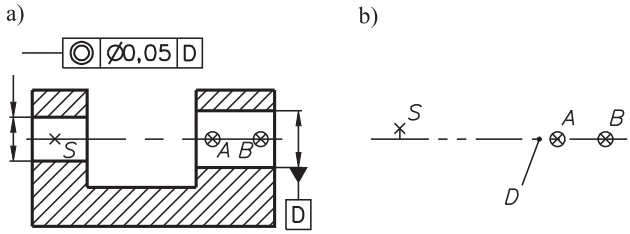

Fig. 2. Measurement of coaxiality deviation: a) a design drawing with characteristic points, b) measurement model

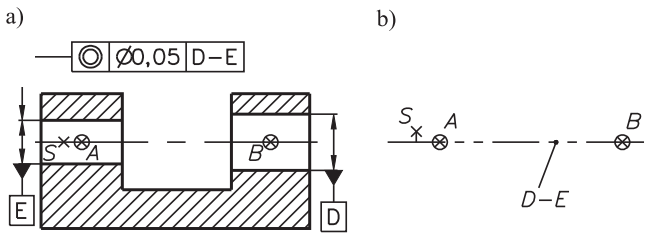

Fig. 3. Measurement of coaxiality deviation from the common axis: a) a design drawing with characteristic points, b) measurement model

In 3 above mentioned tasks, the straight line p is given by two points A and B. Either of the points A or B can be used as the point P in formula (5) and the unit vector v can be calculated as:

$$v = \frac{B - A}{|B - A|} \tag{6}$$

Finally, one gets two formulas for straightness and coaxiality deviations:

$$l_1 = \sqrt{\left(x_{AS} + x_{BA} \cdot t\right)^2 + \left(y_{AS} + y_{BA} \cdot t\right)^2 + \left(z_{AS} + z_{BA} \cdot t\right)^2} \tag{7}$$

where:

$$t = -\frac{x_{AS} x_{BA} + y_{AS} y_{BA} + z_{AS} z_{BA}}{x_{BA}^2 + y_{BA}^2 + z_{BA}^2} \tag{8}$$

and

$$l_2 = \sqrt{\left(x_{BS} + x_{BA} \cdot t\right)^2 + \left(y_{BS} + y_{BA} \cdot t\right)^2 + \left(z_{BS} + z_{BA} \cdot t\right)^2} \tag{9}$$

where:

$$t = -\frac{x_{BS} x_{BA} + y_{BS} y_{BA} + z_{BS} z_{BA}}{x_{BA}^2 + y_{BA}^2 + z_{BA}^2} \tag{10}$$

The uncertainty of measurement is the smaller value of two uncertainties calculated basing on above formulas:

$$u = \min\{u_{l1}, u_{l2}\} \tag{11}$$

4. Model of the measurement of parallelism deviation of axes in space

The appropriate model for the parallelism deviation of axes in space is shown on Fig. 4. The minimal number of points required to determine the deviation is 4. The datum axis is represented by two points *A* and *B*. The axis, direction of which is toleranced, is given by points *K* and *S*. The measuring task is to determine the distance *l* of a point *S* from the straight line parallel to the axis *AB* and going though point *K*.

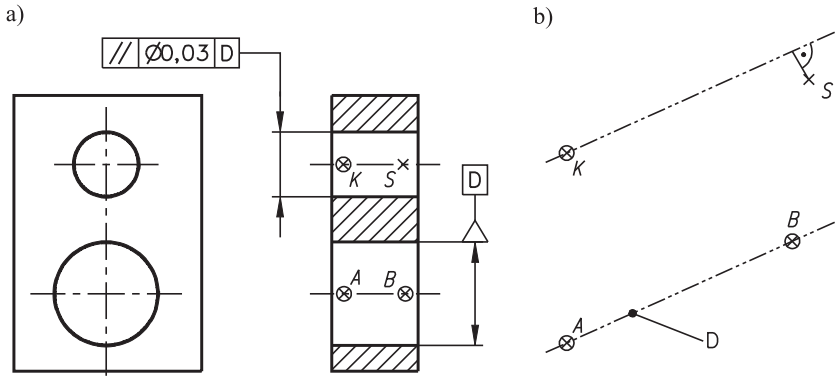

Fig. 4. Measurement of parallelism deviation of axes in space: a) a design drawing with characteristic points, b) measurement model

In the task, the straight line *p* from the formula (5) is parallel to straight line *AB* and goes through point *K*. The unit vector *v* can be calculated as above.

Finally, the formula for parallelism deviation of axes in case of cylindrical tolerance zone is:

$$l = \frac{\sqrt{\left(y_{KS} \cdot z_{BA} - y_{BA} \cdot z_{KS}\right)^2 + \left(x_{BA} \cdot z_{KS} - x_{KS} \cdot z_{BA}\right)^2 + \left(x_{KS} \cdot y_{BA} - x_{BA} \cdot y_{KS}\right)^2}}{\sqrt{x_{BA}^2 + y_{BA}^2 + z_{BA}^2}} \quad (12)$$

5. Model of the measurement of perpendicularity deviation of axis to plane

The appropriate model for the perpendicularity deviation of axis to plane is shown on Fig. 5. The minimal number of points required to determine the deviation is 5. The datum plane is given by three points *A*, *B* and *C*. The axis, the direction of which is toleranced, is given by points *K* and *S*. The measuring task is to determine the distance *l* of the point *S* from the straight line perpendicular to the plane *ABC* and going though the point *K*.

In the task, the straight line *p* from the formula (5) is perpendicular to the plane *ABC* and goes through point *K*. The unit vector *v* (normal vector of the plane *ABC*) can be calculated as:

$$v = \frac{AB \times AC}{\left|AB \times AC\right|} \quad (13)$$

a) b)

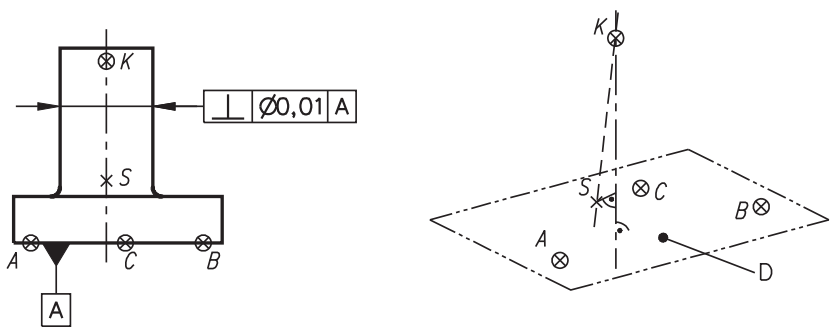

Fig. 5. Measurement of perpendicularity deviation of axis to plane: a) a design drawing with characteristic points, b) measurement model

Finally, the formula for perpendicularity deviation of an axis to a plane is:

$$l = \frac{ax_{KS} + by_{KS} + cz_{KS}}{m} \tag{14}$$

where:

$$\begin{aligned} a &= y_{BA}z_{CA} - z_{BA}y_{CA} \\ b &= z_{BA}x_{CA} - x_{BA}z_{CA} \\ c &= x_{BA}y_{CA} - y_{BA}x_{CA} \\ m &= \sqrt{a^2 + b^2 + c^2} \end{aligned} \tag{15}$$

6. Conclusion

The use of the minimal number of points in the model of measurement of geometrical deviations and expressing the deviation as a function of differences of points coordinates enables analytical estimation of coordinate measurement's uncertainty in accordance with GUM [11] requirements. The proposed methodology can be an alternative to the simulation technique which uses the same data.

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