AN ANALYSIS OF NORMAL CONTACT DEFORMATIONS IN THE BASIC MODEL OF A ROLLER GUIDEWAY OF MACHINE TOOL

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S u m m a r y

This paper analyses the basic model of a roller guideway of a machine tool. It shows that the normal contact deformations in such a model, obtained by the empirical formula given in the manufacturer's catalogue of these guideways, and also by the Hertzian theory, are far smaller than the ones obtained experimentally for such a system. The main reason for these discrepancies seems to be the fact that the basic model and calculation formulae did not take into account the influence of surface roughness of the raceways and roller. It is demonstrated that by using the FE system ANSYS and taking into account the contact stiffness of surfaces roughness, it is possible to get calculation results that are in good agreement with the experimental results.

Keywords: roller guideways, contact deformations, modeling, calculations, experimental tests

Badania kontaktowych odkształceń normalnych w bazowym modelu rolkowej prowadnicy obrabiarki

S t r e s z c z e n i e

W pracy przedstawiono analizę modelu bazowego rolkowej prowadnicy obrabiarki. Wykazano, że odkształcenia kontaktowe normalne w takim modelu określone na podstawie zależności empirycznej, ustalone przez producenta tych prowadnic, a także według teorii Hertza, mają znacznie mniejsze wartości od wyznaczonych doświadczalnie dla takiego układu. Stwierdzono, że główną przyczyną różnicy tych wartości jest brak uwzględnienia w przyjętym modelu i obliczeniach wpływu chropowatości powierzchni bieżni i rolki. Wykazano, że zastosowanie do obliczeń programu MES ANSYS i uwzględnienie sztywności kontaktowej chropowatych powierzchni umożliwia uzyskanie wyników w dobrej zgodności z wynikami badań doświadczalnych.

Słowa kluczowe: prowadnice rolkowe, odkształcenia kontaktowe, modelowanie, obliczenia, badania doświadczalne

1. Introduction

Modeling and analysis of body systems of machine tools, especially including their nonlinearities, require a good knowledge of the mechanical characteristics not only of their components but also of the interface joints

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between them. Particularly great influences on the static and dynamic behaviour of the whole mechanical system of a machine tool have the contact phenomena (contact pressure and deformation, friction, damping and wear), which take place at the guideway connections [1, 2]. That is why studies of these contact phenomena and the development of their modeling and calculating methods have recently gained importance both from the theoretical and engineering practical point of view [3, 4].

In modern precise machine tools rolling guideways (with rollers or balls) are now very often used. Besides their many indisputable advantages (low friction resistance, the lack of stick-slip phenomena), they also have some disadvantages. These include low contact stiffness and low vibration damping. Our earlier investigations of the normal contact deformations at the roller guideways [5, 6], carried out in laboratory conditions and on a real object (a grinding machine 16A/400, manufactured by KARSTENS) have shown great quantitative differences between the results obtained from measurements and from calculations by using the formulae given in the manufacturer's catalogue of these guideways (INA HYDREL [7]). The contact deformations values obtained by measurements were in each case much greater than the ones obtained by calculations.

Analyzing the reasons of the great discrepancy between these results we have concluded, that they are caused by two simplified assumptions until now commonly made in modeling and calculating of these roller guideways. These assumptions are: (1) all rollers have exactly the same diameter and that the force acting at the guideway is distributed evenly on all of them; (2) the surfaces of the rollers and guideway races being in contact are ideally smooth. In real roller guideways of machine tools these assumptions are not fulfilled and the errors caused by these assumptions – as was shown in our study [5, 6] – can be relatively big. The influence of diameters' differences of the individual rollers, which are held within the range of specified manufacturing tolerance, on the contact deformations at the roller guideways, has been studied and described earlier in the paper [6]. That is why in this paper the attention is concentrated on the analysis and evaluation of the calculation methods of the contact deformations in a simple system with one roller.

The main goal of this paper is to show the role and to obtain the quantitative influence of the surface roughness of the roller and the raceways on the distributions of contact pressures and deformations in a commonly used basic calculation model of such guideways (i.e. in a system with only one roller).

2. The subject and goal of investigation

The subjects of investigations described in this paper are normal contact deformations taking place in a linear roller guideway of machine tool, as shown

in Fig. 1a. A basic calculation model for such a roller guideway, recommended generally in the literature [7, 8], is shown in Fig 1b. It consists of one roller (with a finite length *L*) and two flat elements representing the lower and upper raceways of the guideway and pressing on the roller.

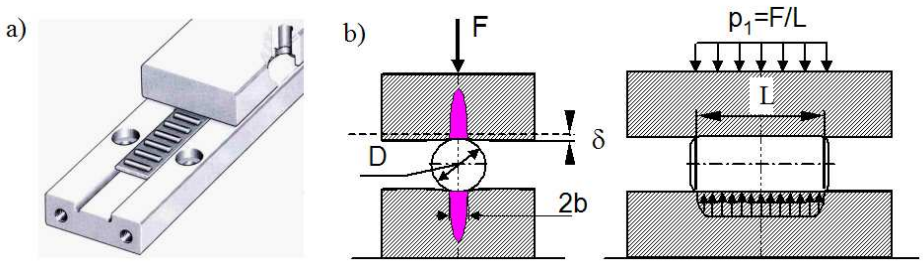

Fig. 1. Linear roller guideway of a machine tool (a) and its basic calculation model (b)

The normal contact deformations of the machine tool rolling guideways, according to their manufacturer's catalogue [7] and also the Hertzian theory [8], are defined by the mutual approach of the two elements pressing on the roller, measured in points remote from their direct contact area with the roller. This mutual approach is a result of the contact and bulk deformations of the roller and the contact deformations of the two raceways pressing on it.

The detailed goal of the investigations presented in this paper is:

1) to determine characteristics of the contact deformations for the system shown in Fig. 1b, using the empirical formula taken from the guideway manufacturer's catalogue [7] and the formula based on the Hertzian theory (taken form the handbook [8]) and compare them with the characteristics obtained from experimental measurements. These characteristics remarkably differ quantitatively from each other,

2) to show that by using the finite element method (FEM) and taking into consideration the surface roughness of contacting elements, it is possible to get calculation results that are in a good agreement with the experimental results.

3. Contact deformation characteristics obtained analytically and experimentally

The empirical formula for calculations of the normal contact deformations in linear roller guideways of machine tools, recommended in their manufacturer's catalogue [7], has the following general form:

$$\delta = K \frac{\left(F/Z\right)^{0,838}}{L^{0,605}} \tag{1}$$

where: δ – denotes the contact deformation, expressed in μm, F – the loading force, expressed in N, Z – the number of rollers (in one row, Fig. 1a), L – the length of the rolling elements, in mm; K – is a coefficient depending on the guideways' shape; its value for the case under consideration (according to [7]) is $K = 0.087$.

The ratio F/Z in formula (1) denotes the mean value of the load acting on one roller, and thus the calculations of the whole guideway connection under consideration (Fig. 1a) can be replaced by a simplified model with one roller, as shown in Fig. 1b.

The formulae based on the Hertzian theory for calculations of the normal contact deformations and pressures, in a system as in Fig. 1b, can be found in the handbook [8]. These formulae have the following form:

$$\delta = 4p_1\left(\frac{1-\nu^2}{\pi E}\right)\left(\ln\frac{\pi EL}{p_1\left(1-\nu^2\right)}\right) \tag{2}$$

$$p(x) = p_o\sqrt{1-\frac{x^2}{b^2}} \tag{3}$$

where: p_1 – denotes the load per unite length of the roller ($p_1 = F/L$), p_o – the maximum value of the pressure in contact, $p(x)$ – the pressures in the contact area as a function of x $\left(-b \le x \le b\right)$, b – half width of the contact area (according to the Hertzian theory, Fig. 1b), which can be calculated by:

$$p_o = 0,418\sqrt{\frac{FE}{RL}} \tag{4}$$

$$b = 1,08\sqrt{\frac{FD}{LE}} \tag{5}$$

where E and ν – are the material constants of the system components, under the assumption that they are made from the same material.

Numerical calculations using formulae (1) and (2) were carried out with the following assumptions: Z = 1, L = 13,8 mm, K = 0,0867, $E = 2\times10^5$ MPa, ν = 0,3, D = 3 mm. The value of the force F was increasing stepwise by 10 N in the range of 0 to 100 N. The obtained characteristics, expressed by δ = f(F), are presented in Fig. 2 by the curves 1 and 2, respectively.

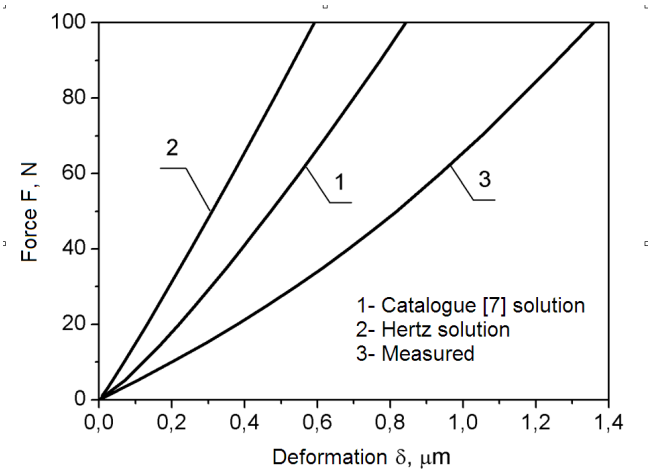

Fig. 2. Characteristics of the normal contact deformations, δ = f(F), obtained from the calculations (curves 1 and 2) and experimental measurements (curve 3)

The curve 3 in Fig. 2 represents the characteristic of the contact deformations for the case under consideration derived from the experimental results obtained earlier and described in detail in [6]. The curves 1 and 2 presented in Fig. 2, obtained by the formulae (1) and (2), recommended in the literature and applied in the engineering practice, differ noticeably from each other and in both cases the values of the calculated deformations are much smaller than the experimental results. An explanation of these discrepancies is possible, if the model of the system under the investigation is extended to include the influence of the surface roughness of the contacting elements (i.e. the raceways and the roller).

4. Calculations by the FEM without taking into account the surface roughness

Nowadays, calculations of stresses and deformations in machine components and systems are ever more often carried out by means of the finite element method (FEM). The modern FEM programs not only can handle separate machine elements but also contact problems for complicated multi-body

systems with unilateral constrains by taking into account friction forces and contact stiffness [3, 4, 9-13] – features very attractive for the current investigation. The finite element (FE) system ANSYS v. 10 will be used in this work.

In order to simplify the calculations and compare the results with the ones obtained earlier for the model shown in Fig. 1b by means of formulae (1) and (2), the first FE-model reflects the assumptions of the Hertzian theory, i.e. that in the system there is a plane (2D) state of strain, the surfaces of the interacting elements are ideally smooth and in the contact between them there are no friction forces. A more complicated FE-model accounting for the surface roughness will be presented later in this paper.

Taking into account the symmetry of the system under consideration (Fig. 3a), only a quarter of it was modeled in the FE program. This model is shown in Fig. 3b. The mesh of the FE is refined in the contact region to capture the complicated variations of contact pressures and deformations. For modeling of solids, 3D finite elements SOLID45 were used. Plane state of strain was assumed in the system. For modeling of the contact region surface–to-surface contact elements with 8-nodes CONTA 174 and TARGE 170 were used. Computations were carried out using the *Lagrange multiplier method*, which does not take into account the contact stiffness and the penetration of the surfaces acting on each other.

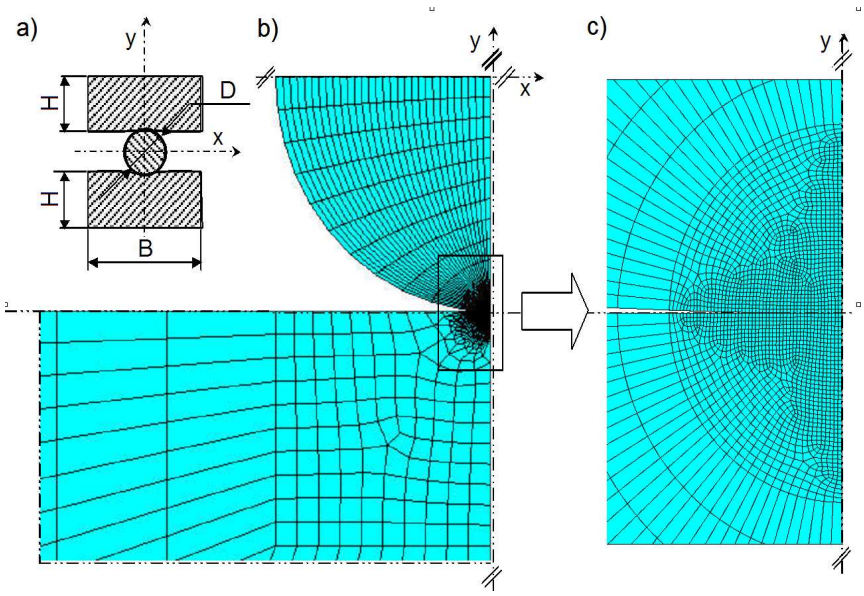

Fig. 3. Schema of the considered system (a) and its FE-model (b, c)

From the symmetry of the modeled system (Fig. 3a) follows that its horizontal and vertical sections along the x and y axes remain flat after applying the load. Therefore the vertical displacements of all the points (nodes) lying on the top surface of the FE model (Fig 3b) will be equal, whereas displacements of these points within the horizontal plane are possible in the x direction only and can have different values. The constructed FE model takes into consideration all these constrains. The vertical displacement $w_o = w_{max} = w_y(0)$ of the top surface in the EF model (Fig. 3b), according to the definition taken earlier, is the measure of the contact deformation of the subsystem, which represents only one half of the total contact deformation (δ) of the whole system under investigation (see in Fig. 3a).

The displacement w_o is the result of the contact and bulk (material) deformations of the roller as well as the contact deformation of the elastic raceway, which the roller is pressing on. The raceway, represented by the flat element, is resting on a rigid foundation and is restrained from movement in the x and y directions.

The numerical calculations were carried out with the following assumptions: D = 3 mm, $E_1 = E_2 = 2\ 10^5$ N/mm^2, $\nu_1 = \nu_2$ = 0,3, L = 13,8 mm, μ = 0,3, by taking the loading force $F_1 = 0{,}5F$. The force F was increasing stepwise by 10 N in the range of 0 to 100 N. For each value of the force F the state of stress and strain in the system was obtained. The calculation results are shown in Figures 4-6.

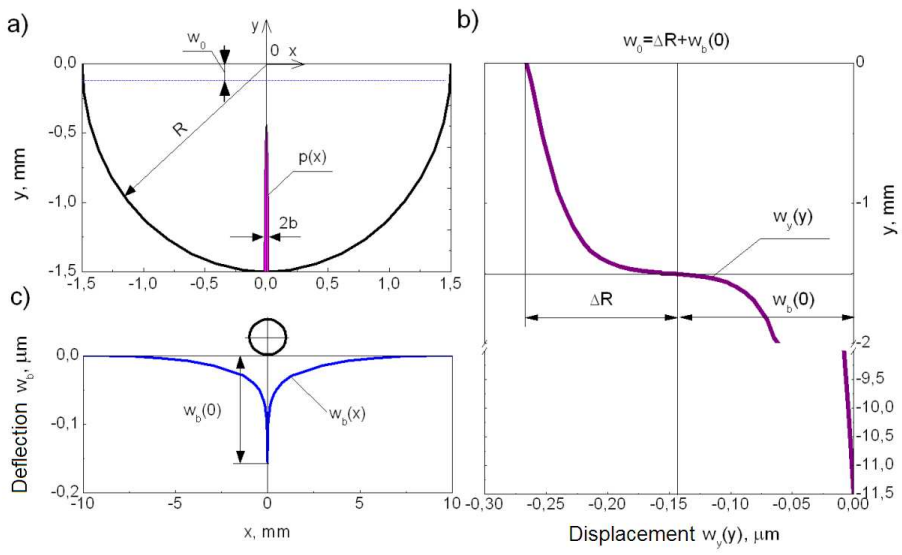

Fig. 4. Numerical results obtained by the FE-system ANSYS for the loading force F = 100 N: a) schema of the system, b) distribution of the vertical displacements for the points lying on the y axis, c) deflection line of the raceway

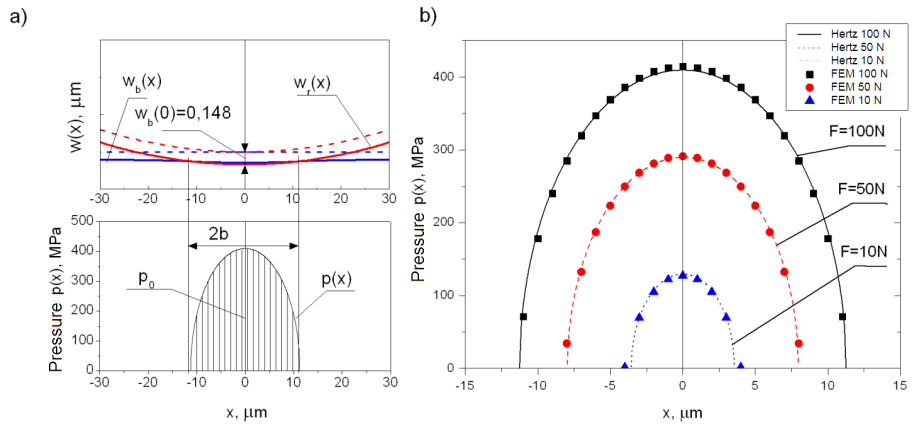

Fig. 5. Distributions of displacement and pressure at the contact area: a) deflection curve of the raceway, $w_b(x)$, the width of the contact area, $2b$, and the pressure distribution p(x), b) comparison of the results obtained by means of the FEM and Hertzian theory for three values of the loading force F

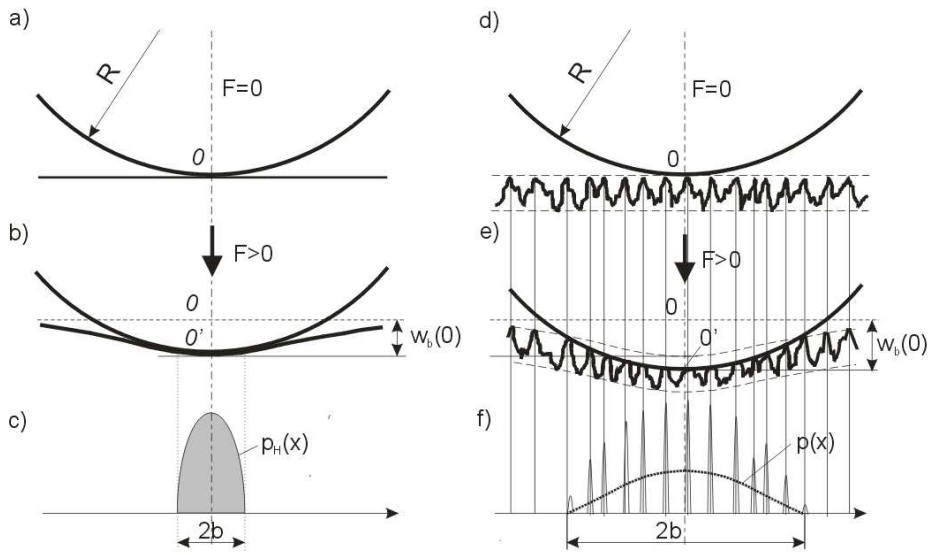

Fig. 6. Schemas of a contact between a smooth sphere (or cylinder) and a plane, with an ideally smooth surface (a, b, c) and with a rough surface (d, e, f)

Figure 4a represents the bottom half of the whole system under investigation. The graph in Fig. 4b shows the vertical displacements of the points lying at the y axis, and Fig. 4c – the deflection curve of the raceway as a function of x. A closer examination of the graph in Fig. 4b shows contributions

to the total displacement w_o of the upper surface in the FE model (Fig. 3b) attributed to the deformation of the roller (ΔR) and to the deformation of the raceway ($w_b(0)$). The contact deformations of the raceway $w_b(0)$ have a slightly larger influence on the total displacement w_o than the contact and bulk deformations of the roller (ΔR).

Figure 5a shows the vertical displacements of the raceway, the width of the contact area between the roller and the raceway, and the contact pressure distribution at this area, obtained by the FEM for the maximum value of the loading force (F = 100 N). These numerical results are compared with the analytical results based on the Hertzian formulae. For the case under investigation, an elastic cylinder in contact with an elastic plane, the contact pressure distribution can be calculated by the formula [8]:

$$p(x) = p_o\sqrt{1-\frac{x^2}{b^2}} \tag{6}$$

where: b denotes the half width of the contact area (Fig. 5a), and p_o – the maximum value of pressure at the contact, which can be given as:

$$p_o = 0,418\sqrt{\frac{PE}{RL}} \tag{7}$$

For comparison purposes, the calculations by means of the FEM and Hertzian formulae were carried out for three values of the loading force: F = 100, 50 and 10 N. The calculation results are shown in Fig. 5b. In each case the numerical results match well the analytical results for both the distribution and the value of the contact pressures. A great advantage of the FEM is that it allows to calculate and to analyze the stresses and deformations more general and more in depth, not only in the contact region, but also in the whole system, and covers a greater range of contact problems, both of local and global character.

5. Calculation by means of FEM with taking into account the surface roughness

When applying the Hertzian analysis, and very often the FEM as well, to investigate contact problems, the assumption is made that the interacting surfaces of the contacting bodies are topographically ideally smooth. In reality, machine elements have more or less rough surfaces and, even after being precisely machined, they do not satisfy the Hertzian assumptions. The roughness of a real surface depends on the machining process and, in general, the

roughness parameter R_a can have a value in the range of 0.2-10 μm, and after a fine grinding operation, R_a can be in the range of 0.2-1 μm.

Due to the roughness of the surface the contact between such surfaces is, in general, discontinuous. Rough surfaces come into contact only at isolated points where the asperities on the two mating surfaces come together. The real contact area is only a small fraction of the nominal contact area. This is equally true whether the apparent contact area is macroscopic, as in the contact of two nominally flat solids, or microscopic, as in the contact of two spheres [14].

A geometrical illustration of a contact between a sphere or cylinder and a plane surface is given in Fig. 6. Figures 6a, b, c preset the contact of smooth surfaces, and Figs 6d, e, f – the contact of rough surfaces. For the sake of clarity, the surface roughness in Figs 6d, e, f has been transformed to the plane surface [12-15] that is in contact with a smooth surface of a sphere or cylinder of radius *R*.

In a concentrated contact of rough surfaces (Figs 6d, e, f) the nominal contact extends over a larger area and the effective pressure distribution is much lower than would be the case if the solids were perfectly smooth (Figs 6a, b, c). The nominal contact area is discontinuous and contains many deformed asperities and these then act as a compliant layer lying over the surfaces of the bodies. Consequently, the measured value of the normal contact deformation is usually remarkably larger than the theoretical value, obtained from the Hertzian theory (for the contact of perfect smooth surfaces). This fact, which is well known from the literature [14, 15], has been confirmed in our experimental studies [6], whose results are shown in Fig. 2

The essential question here is: how great the influence of the surface roughness on the normal contact deformation δ and the distribution of nominal pressure can be, and what does it depend on? Certain qualitative and quantitative analyses of this problem, in the case of a rough plane surface which is in contact with a smooth sphere of radius *R*, where already made earlier. Detailed descriptions and results of these analyses can be found in the literature [9, 10, 12-15]. The obtained results showed that the value of the maximum nominal pressure in the contact of a sphere with a plane rough surface, at lower loads, is about three times less than the maximum pressure calculated by the Hertzian theory (for such a contact of ideally smooth surfaces). Generally, it depends on the material, surface topography (the statistical highs parameters of the roughness), load, and the radius of the sphere [14, 15]. In each case the surface roughness plays a significant role.

The calculation results obtained in the literature [12-15] for the contact of a sphere with a plane surface, cannot be directly applied to the quantitative estimation of the influence of the surface roughness on the contact deformation in the investigated system with a roller, where a plane state of strain is assumed. The roughness of the surface in the contact between a roller and a plane surface plays an identical role as in the contact between a sphere and a plane surface;

and explains entirely the much greater value of the measured contact deformation than the one obtained by the analytical or numerical calculations for the model based on the assumption that the surfaces are ideally smooth. The Hertzian analysis is merely the limiting case to which the real contact deformation approaches as the surfaces become ideally smooth.

From the earlier investigation results [9, 10, 12, 14, 15] follows that the heights parameters of the surface roughness R_a, R_q, R_z and $R_t(R_{max})$, and especially the standard deviation $\sigma = R_q \approx 1{,}25R_a$ have a significant influence on the pressure distribution and contact deformation. Measurements of these parameters for the investigated system gave the following mean values:

- for the roller

$$R_a = 0{,}073\ \mu\text{m},\ R_q = 0{,}097\ \mu\text{m},\ R_z = 0{,}57\ \mu\text{m},\ R_t(R_{max}) = 0{,}83\ \mu\text{m},$$

- for the raceway

$$R_a = 0{,}25\ \mu\text{m},\ R_q = 0.32\ \mu\text{m},\ R_z = 1{,}57\ \mu\text{m},\ R_t(R_{max}) = 2{,}09\ \mu\text{m},$$

- for compressing plats

$$R_a = 0{,}15\ \mu\text{m},\ R_q = 0{,}19\ \mu\text{m},\ R_z = 1{,}00\ \mu\text{m},\ R_t(R_{max}) = 1{,}33\ \mu\text{m}.$$

The obtained roughness parameters of the tested surfaces have the same order of magnitude as the measured values of the contact deformations δ. Thus, one can assume that the surface roughness has a significant influence on the distributions of the contact pressures and deformations in the investigated system.

For the numerical analysis of the assumed basic model of the roller guideway, by taking into account the contact stiffness of the rough surfaces, the same nominal FE model is used as in the case with the ideally smooth surfaces (Fig. 4). The difference is in the method selected to carry out the calculations. The earlier applied calculation method, i.e. the *pure Lagrange method*, concerned the contact of ideal smooth surfaces. It did not take into account the mutual penetration of the rough surfaces and did not require the knowledge of the contact stiffness. The presently used *Augmented Lagrange method* takes into account the contact stiffness coefficients and the mutual penetration coefficient. In this method there are generally two coefficients of contact stiffness: *FKN* – in the normal direction, and *FKT* – in the tangential direction. The allowed mutual penetration of the rough surfaces is determined by the coefficient *FTOLN*. Unfortunately, there is no direct relationship between these coefficients and the parameters of the surface roughness; they have only a general (purely mathematical) sense in the modeling of flexible contact joints.

To determine the contact deformations δ and the nominal pressures distributions $p(x)$ as a function of the load F and the contact stiffness *FKN*, calculations are carried out for the FE model as in Fig. 4 and the same set of general data as before (for the ideally smooth surfaces). Several values, from the range of 0.1 to 100, of the contact stiffness coefficient *FKN* are selected. For each of the assumed *FKN* coefficient value, the value of the loading force F is increased stepwise by 10N in the range 0-100 N. The *Augmented Lagrange method* is used. The obtained results are presented in Figures 7 and 8.

Figure 7 shows characteristics of the contact deformation δ = f(F) for different values of the coefficient of contact stiffness *FKN*. The triangular marks represent the results obtained by the Hertzian theory (formula (2) and the square marks – the experimental results (presented before in Figure 2). The large values of the *FKN* coefficient correspond to the smooth (without roughness) surfaces. The calculation results for *FKN* = 0,152 are in a very good agreement with the experimental results. Thus, the *FKN* coefficient value of 0.152 corresponds to the roughness parameters of the experimentally tested system [6]. The measured values of the roughness parameters of these elements are given above.

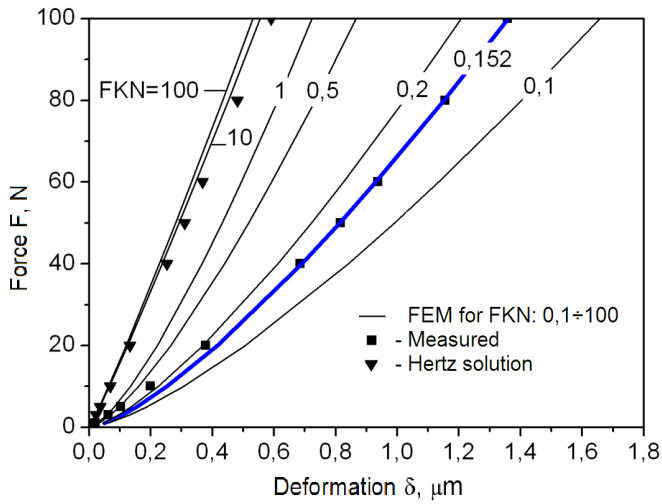

Fig. 7 Characteristics of contact deformations obtained by means of numerical calculations (for different values of *FKN*), analytical calculations (using the Hertzian formula) and experimental measurements

Figure 8 shows the pressure distributions in the contact zone between the roller and the raceway for different values of the contact stiffness coefficient *FKN*. The results obtained for *FKN* = 100 are in a good agreement with the Hertzian values, which concern the contact of ideal smooth surfaces. The surface roughness causes an enlargement of the nominal contact area and a decrease of

the pressure. For the value *FKN* = 0.152, corresponding to the experimentally investigated system, for which the contact deformations were obtained by measuring, the maximum pressure is about three times lower than the maximum pressure obtained by the Hertzian theory. This result is comparable with the results presented in the literature [14, 15] for the contact between a smooth sphere and a rough plane surface.

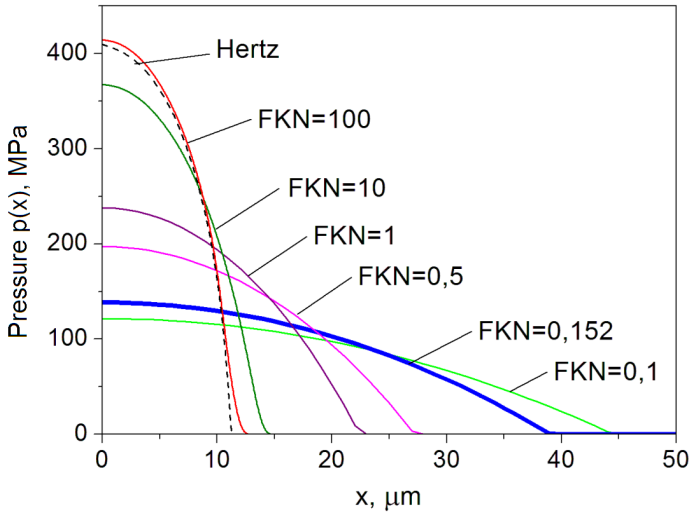

Fig. 8. Contact pressure distributions obtained by means of numerical calculations for various values of *FKN* and by analytical calculations using the Hertzian formula

Conclusions

The contact stiffness of guideway systems plays an important role in modern modeling and dynamic analysis of supporting systems of machine tools, particularly when considered from the non-linear point of view. As this paper shows, the formulae provided in the manufacturer's catalogue [7] for the calculations of the contact deformations in the investigated roller guideways are inaccurate and give results biased with significant errors. Therefore, it seems necessary to conduct a more comprehensive verification and correction of these formulae.

Also the classical Hertzian formulae cannot be used to calculate the contact pressures and deformations in such guideways because they relate to the contact between ideally smooth surfaces. In the investigated roller guideways the surfaces of the raceways and rollers are in reality rough and do not fulfill this

assumption. The heights parameters of the roughness of the raceways are of the same order of magnitude as the measured contact deformations and they play a significant role in the analysis of contact pressures and stiffness in such guideways.

It has been shown that using the FE-system ANSYS and taking into consideration the contact stiffness of the interacting bodies it is possible to calculate contact deformations that are in a good agreement with the measured values.

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