

# THE FLEXSPLINE OF THE HARMONIC DRIVE WITH DIFFERENT TOOTH PROFILE

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## Summary

This article presents the process of determining reduced stress which occurs during the deformation of the flexspline of the toothed harmonic drive. The discussed issue is complex as the stress distribution results from the deformation of the flexspline by the generator and from the load it carries. The article takes into account flexsplines of various tooth profiles: involute, rectilinear and convexo-concave (BBW). The flexspline was deformed by the double-roll generator. To conduct numerical calculations ADINA software, based on the Finite Element Method (FEM), was used. The calculation results for each tooth profile were put together for comparison and evaluation.

**Keywords:** harmonic drive, flexspline, FEM, ADINA

## Koło podatne przekładni falowej o różnych zarysach zębów

### Streszczenie

W pracy zaprezentowano proces wyznaczania naprężeń zredukowanych powstających w odkształcanym kole podatnym falowej przekładni zębatej. Dotyczy to złożonego zagadnienia – występujące naprężenia są skutkiem zarówno odkształceń koła na skutek działania generatora, jak również przenieszonego obciążenia. W pracy uwzględniono koła o różnym zarysie zębów: ewolwentowym, prostoliniowym i kołowo-łukowym (BBW). Koło podatne poddano odkształceniu przy użyciu generatora dwurołkowego. Do obliczeń numerycznych przyjęto metodę elementów skończonych (MES) i program ADINA. Przeprowadzono analizę porównawczą wyników obliczeń dla różnych zarysów zębów. Wyniki obliczeń otrzymanych dla różnych zarysów zębów zostały zestawione ze sobą w celu ich porównania i oceny.

**Słowa kluczowe:** zębata przekładnia falowa, koło podatne, MES, ADINA

## Introduction

In harmonic drives (Fig. 1) the transmission of movement occurs as the result of the travelling of the deformed wave of one of the elastic elements of the drive towards the other element. That deforming element is the flexspline (Fig. 2) which, in the areas supported by the generator, meshes with the internally toothed circular spline [1, 2]. Similar to the planetary drive, there is a multi-stream

transmission of energy in the harmonic drive. What is more, they can work under long-term one-direction load as well as under short-term reversible load. In such difficult operating conditions the multi-pair internal meshing takes place. This causes great difficulties in calculations as a large number of input data needs be taken into account, which is difficult to describe and analyse by means of traditional calculation methods [3, 4]. Due to the fact that the structure and the character of the operation of the drive is very complicated, the simplified model of the Finite Element Method (FEM) was used for numerical strength calculations [5, 6]. The simplification meant the use of the 2-D models which were cross-sections that went through the centre of the width of the toothed rims of both gears and the generator. Due to the fact that the two-wave cam generator was used, which resulted in the symmetry of the deformation and the load, it was also enough to take into account just a half of the perimeter of the flexspline.

Geometrical and strength calculations allowed to determine the parameters needed for drawing the profiles of the flexspline and the circular spline. The models of the undeformed flexspline, the circular spline and the cam generator were prepared in SolidWorks software and used for analysis.

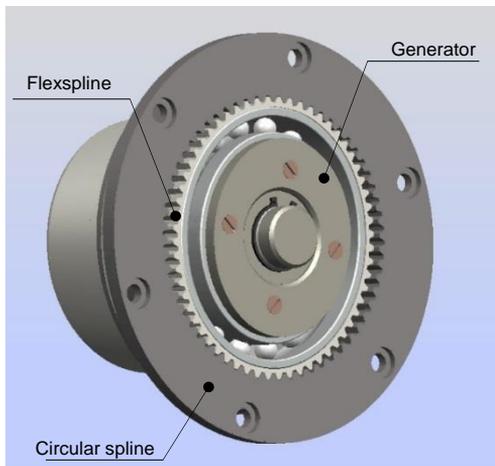


Fig. 1. The model of the single-stage harmonic drive with the cam generator

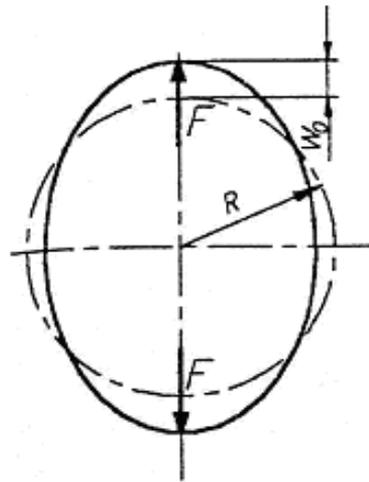


Fig. 2. The form of the flexspline of the toothed harmonic drive deformed by the double-roll generator

Three models of the drive were prepared for the calculations. The models differed from one another only by the tooth profile [7-9]. As the basis for the calculations the involute profile was used which, due to the technology of the steel gear manufacturing, is most common. The second, alternative, profile used for the calculations was the convexo-concave (BBW) profile which, in classic toothed

drives, is used to reduce contact stress. The third solution was the rectilinear profile, which is easy to model in the parametric CAD programs. Such approximation of the profile is acceptable as large diameters and a great number of teeth, in relation to a very small module, make the curvature of the involute on the tooth flank barely noticeable. Figure 3 shows the simplified models of the flexspline and the circular spline of the involute tooth profile. The operation of the wave generator was simulated by the radial transmission on the major axis of the generator.

## 1. The calculation model for the harmonic drive

The strength calculations for the flexspline were made in ADINA software with the use of the Finite Element Method. To determine the stress distribution the flat model of the flexspline, prepared in the external program and imported in the IGES format, was used. Next, the models for all three calculations were prepared in the same way. The models underwent the process of discretization with the use of the quadrangular linear finite elements.

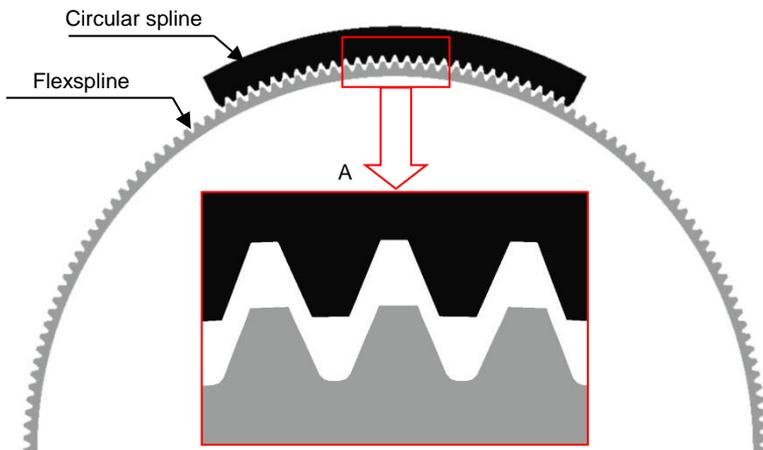


Fig. 3. The simplified model of the flexspline and the circular spline made in SolidWorks

In order to follow the actual mating of the main elements of the analysed harmonic drive, the contact areas were defined in places where gears touch each other. The contact areas are clearly visible in a close-up in Fig. 4. Due to the use of the simplified models, the geometrical bonds were introduced for the elements on the minor axis of the flexspline, which allowed them to move only in a radial direction (horizontally). The deformation of the flexspline by the generator was

obtained by the radial node transmission on a major axis of the generator by the value previously determined in the calculations.

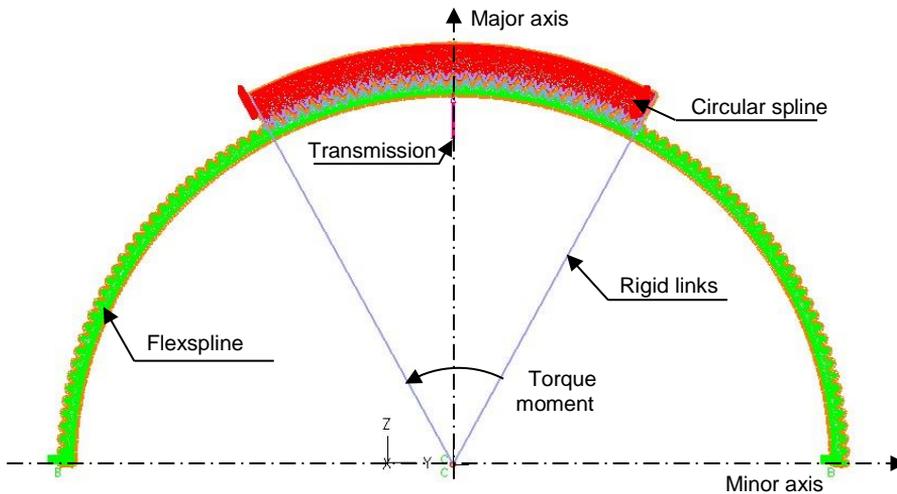


Fig. 4. The simplified calculation model of the harmonic drive prepared for calculations in ADINA software

By using so called rigid links in the model, the circular spline was fixed in its centre with the possibility to rotate only around it. In the analysed case the circular spline was loaded with a torque moment of 200 Nm. For the models prepared this way the calculations were made for all three tooth profiles consecutively: involute, rectilinear and convexo-concave (BBW).

## 2. The calculation results

The calculations of all three tooth profiles were made without any problems and the results were obtained for each model in the form of the reduced stress distribution. As the presentation of the entire gear models for the three cases would be very extensive, the article presents the results for the chosen characteristic areas with the description of the remaining results.

The main aim of the calculations was to determine the impact of the tooth profile on the gear stress with the actual operating conditions being taken into account. Therefore, not only was the flexspline deformed but also loaded with the moment obtained by the mating circular spline. The solutions for all three calculation cases were put together. Thanks to the use of contact elements it was possible to observe the contact stress caused by the engagement of the teeth of

both gears. What is more, tooth engagement causes their bending and the diversification of the stress values at the root on both sides of the tooth.

Figures from 5 to 7 present the stress distribution for key fragments of gears of the harmonic drive under load, which were obtained in the calculations. There is a clear difference in the stress distribution for the involute and rectilinear profiles. The solution presented in Fig. 7 shows the engagement on the entire height of teeth in subsequent meshing pairs. For the rectilinear profile the tooth flanks of the flexspline mesh only with the tips of the circular spline and that disqualifies the profile for commercial use.

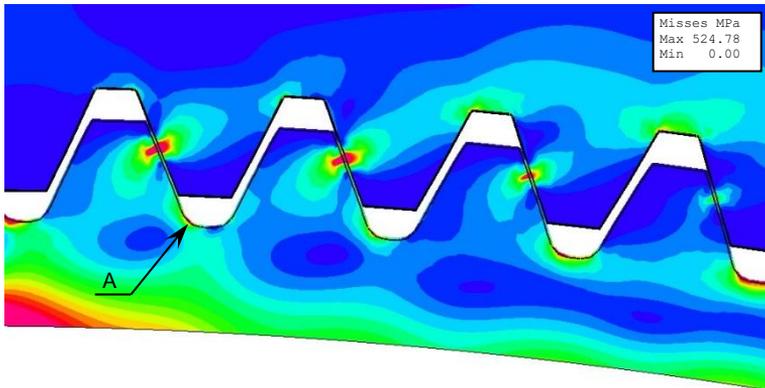


Fig. 5. The reduced stress distribution MPa in gears with the involute tooth profile

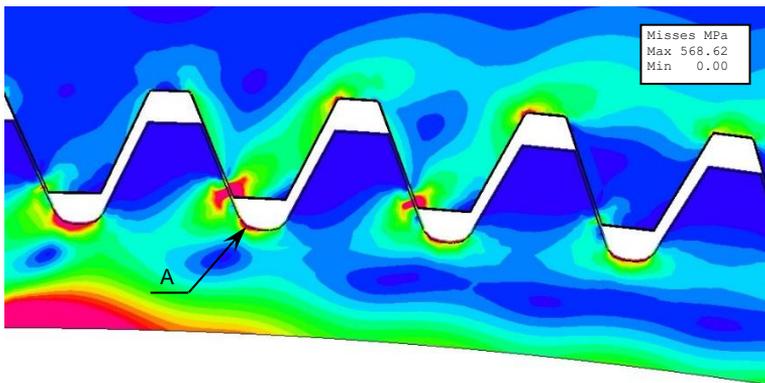


Fig. 6. The reduced stress distribution in gears with the rectilinear tooth profile

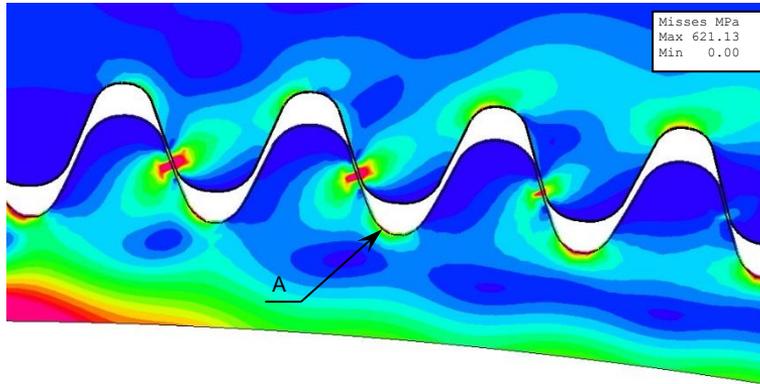


Fig. 7. The reduced stress distribution in gears with the convexo-concave (BBW) profile

The maximum values of contact stress for the involute profile are 524.78 MPa and for the rectilinear profile are 568.42 MPa. The highest values of all tested calculation models were calculated for the convexo-concave profile and are as high as 621.13 MPa.

To observe the impact of the load on the stress in the flexspline even more closely, the diagrams of the bending stress at the root of the subsequent teeth, in places marked as “A” in Fig. 5 and 7, were prepared. The diagram for the flexspline with the involute tooth profile is presented in Fig. 8 and for the convexo-concave profile in Fig. 9. The model of flexspline with the rectilinear tooth profile was not analyzed due to the most unfavorable stress distribution. The maximum stress values, higher by about 10%, were calculated for the convexo-concave profiles, similar as for the gear under no load, which is caused by a slightly thicker sleeve of the flexspline.

The diagram was not prepared for the drive with gears of rectilinear tooth profile, as the profile showed very unfavourable meshing conditions and was rejected. Thanks to the prepared diagrams we can see that the stress values as well as the course of the diagram changed after putting a load on the drive. The diagram is not symmetric to the major axis of the generator ( $0^\circ$  angle) but shifted towards the load that works clockwise. This means that, out of all tooth pairs that mesh, their bigger number is on the right side of the major axis of the generator and it is there where the teeth are under the heaviest load.

Also, we can notice that, in the case of the convexo-concave profile, the change of the maximum stress values was considerably smaller than in the case of the involute profile after putting a load on the drive. This proves that the convexo-concave profile is not only characterised by better meshing conditions on the tooth flanks but it is also less sensitive to the changes of the load value. However, in general, the highest stress values for the calculation model were obtained for this profile.

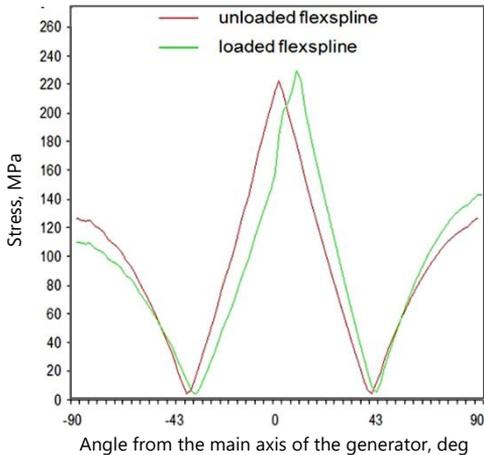


Fig. 8. Stress at the root of the subsequent teeth with the involute profile of the flexspline deformed by the double-roll wave generator

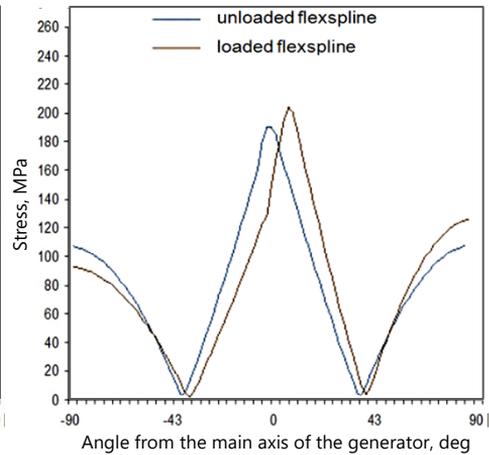


Fig. 9. Stress at the root of the subsequent teeth with the convexo-concave profile of the flexspline deformed by the double-roll wave generator

### 3. Conclusions

On the basis of the conducted analysis we can conclude that the Finite Element Method (FEM) allows to determine the stress distribution in a very detailed way in the main components of the toothed harmonic drive. With the use of the contact elements we can also determine the character of mating between the flexspline and circular spline and, on the basis of the provided information, we can choose the right geometrical parameters. This led to the rejection of the solution with the rectilinear tooth profile, as it caused the incorrect meshing which could have led to the damage of the teeth and consequently the entire drive.

As the result of the applied load there are changes in the stress distribution in relation to the stress in the flexspline deformed only by the generator. When under the load, the flexspline deforms unsymmetrically to the major axis of the generator for all analysed profiles.

For the involute profile the maximum contact stress on the tooth flanks of the harmonic drive is over 520 MPa, whereas for the convexo-concave profile it is considerably higher and exceed the level of 600 MPa. High levels of bending stress also appear on transition radius at the root of the tooth - both in the flexspline and in the circular spline, but their values are definitely lower than contact stress. In order to reduce the stress values of transition curves at the root of the tooth we can use e.g. larger radius which will cause lower concentration of stress in those places.

The rectilinear profile for the gears made of plastic are not recommended due to their manufacturing technology. The convexo-concave profile is not good for

gears with a large number of teeth and small modules due to the highest values of stress obtained in the analysis. What is more, the cost of their manufacturing is very high. From the conducted analysis of stress it is clear that the involute profile of the harmonic drive is favourable and justifiable. The involute profile is rather simple to make and many companies specialize in manufacturing of gears for the harmonic drives with such profile.

The obtained results are starting material for further work related to designing and testing harmonic drives. The obtained results should be verified in stationary research [10, 11] for the drives presented in the article.

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