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MATHEMATICAL MODEL OF TITANIUM TURNING AND IT'S APPLICATION IN MULTI-CRITERION OPTIMIZATION OF THIS PROCESS

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Summary

The article includes information on basic properties of titanium and its alloys, features for titanium machining, the development of the experiment-based mathematical model for straight turning of the WT3-1 titanium, single- and multi-criterion optimization of the selected process features.

Keywords: titanium, mathematical model, multi-criterion optimization, optimum parameters

Model matematyczny toczenia tytanu i jego zastosowanie do wielokryterialnej optymalizacji procesu

Streszczenie

W pracy zaprezentowano podstawowe właściwości fizyczne i mechaniczne tytanu i jego stopów. Scharakteryzowano podstawy obróbki skrawaniem tytanu. Przedstawiono budowę doświadczalnego modelu matematycznego procesu toczenia wzdłużnego stopu tytanu WT3-1. Podano przykład optymalizacji jedno- i wielokryterialnej złożonego przebiegu procesu skrawania.

Słowa kluczowe: tytan, model matematyczny, optymalizacja wielokryterialna, parametry optymalne

1. Introduction

Research on the titanium turning has been conducted in order to develop a mathematical model due to the random nature of such process. Titanium, being a constructional material, features the following properties [1-4]:

- high level of tensile strength R_m [MPa], both at room and elevated temperature,
- high level of relative strength R_m/ρ,
- relatively low density ρ [g/cm^3],
- low value of thermal conductivity coefficient,
- high resistance to corrosion,
- high plasticity.

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Titanium is typically applied in the aircraft and aerospace industry, shipbuilding, chemical engineering and in medicine. Titanium and titanium alloys are considered hard-to cut materials due to the following features [5, 6]:

- high degree of chemical affinity with the cutting tool edge material,
- particle adherence inclination,
- strength conservation at elevated temperatures,
- low thermal conductivity.

The research has also been aimed at searching for new methods of constituting the properties of superficial layer for machine parts in view of optimization procedure. This process belongs to a category of special techniques because of the output quantity shaping by physical phenomena in various machining conditions.

The up-to-date tool materials, machine tools and manufacturing techniques make it possible to obtain very high temperatures in the cutting zone, up to 1000 degrees Celsius. The cutting fluid which has been introduced into this area evaporates instantly. This is a serious limitation on the low-pressure coolant stream introduction into the cutting tool edge proximity so that the effective cooling becomes impossible. One of the possible solutions is just abandonment of the cooling and carrying out the process at the lowered parameters of cut. Coolant-free machining process has also some advantage from the ecology viewpoint.

2. Experimental investigation

Because of complexity of the course of physical phenomena taking place during the surface formation in the straight turning process, the experimental investigations were carried out according to the experimental design theory [7]. The five-level, composite, rotatable experimental design was developed for three independent variables (Table 1).

The independent variables v_c, f, a_p were used within the following range:

- $v_c \in \langle 10.0 - 40.0 \rangle$ m/min – surface speed (cutting speed),
- $f \in \langle 0.05 - 0.3 \rangle$ mm/rev – feed rate,
- $a_p \in \langle 1.0 - 2.5 \rangle$ mm – depth of cut.

The following designations were used in table I: $\rho = 1.682$ – star radius value, i – experiment No. ($i = 1, 2, 3, \ldots, N$), $N = 20$ – number of the experiments, j – output quantity index ($j = T, R_z, F_c, F_f, F_N, k_c, P_c, Q_v$) where: T [min] tool edge life, R_z [µm] – roughness parameter (the highest asperity value according to the DIN standard), F_c [N] – cutting force, F_f [N] – axial force, F_n [N] – radial force, k_c [N/mm^2] specific cutting force, P_c [kW] – cutting power, Q_v [mm^3/s] – material removal rate (volumetric).

Table 1. Experimental design layout

No	**Parametres**			
	v_c	f	a_p	$Y_{i,j}$
1	-1	-1	-1	$Y_{1,j}$
2	+1	-1	-1	$Y_{2,J,}$
3	-1	+1	-1	$Y_{3,j}$
4	+1	+1	-1	$Y_{4,j}$
5	-1	-1	+1	$Y_{5,j}$
6	+1	-1	+1	Y_{6j}
7	-1	+1	+1	$Y_{7,j}$
8	+1	+1	+1	$Y_{8,j}$
9	$-\rho$	0	0	$Y_{9,j}$
10	$+\rho$	0	0	$Y_{10,j}$
11	0	$-\rho$	0	$Y_{11,j}$
12	0	$+\rho$	0	$Y_{12,j}$
13	0	0	$-\rho$	$Y_{13,j}$
14	0	0	$+\rho$	$Y_{14,j}$
15	0	0	0	$Y_{15,j}$
16	0	0	0	$Y_{16,j}$
17	0	0	0	$Y_{17,j}$
18	0	0	0	$Y_{18,j}$
19	0	0	0	$Y_{19,j}$
20	0	0	0	$Y_{20,j}$

Research of the process output quantities was conducted for the samples made of titanium WT3-1, with diameter Φ = 80 mm and length L = 450 mm. The following mechanical properties have been assumed according to the authorized attestation documents: ultimate tensile strength R_m = 1100 MPa, elongation A_5 = 13%, Brinell hardness 350.

The experiments were conducted on the standard toolmaker's lathe using standard tool-holder hR171.26-2525.1 with the interchangeable triangular inserts TNMG160308 made of tungsten carbide H20, with the cutting edge rounding radius r_n = 0,02 mm. Flank wear VB = 0,3 mm was adopted as a critical measure (criterion) of the cutting tool edge dullness. The wear measurement was performed on the toolroom Zeiss microscope. The height of roughness asperities was determined by the luminous cross-section method. The cutting force components were measured by the extensometric dynamometer.

Since the process output values i.e. cutting force components, the height of the surface roughness summit R_z (DIN), cutting power, specific cutting force are dependent on the cutting edge wear, these values were determined for the mathematical model as average values throughout the tool life period in order to limit a number of experiments in the experimental design. It is in this case connected with the exclusion of machining time t as an independent variable which affects the tool wear. Time influence on the specified output quantities has been taken into consideration by dimensionless relationship of the following shape:

$$\frac{y_i}{y_{i,śr}} = a_{0,i} + \sum_{j=1}^{r} a_{i,j} \left(\frac{t}{T}\right)^r \tag{1}$$

y_i – general designation for the output quantities, $y_{i,śr}$ – general designation for the average value of y_i, a_0 – constant factor in the formula, a_j – polynomial factor (regression factor), i – index for physical interpretation of a given variable y, ($i = T, R_z, F_c, F_f, F_N, k_c, P_c$), t – machining time corresponding to a value of y_i, T – tool life in a given experiment, j – polynomial degree index ($j = 1, 2, .., r$).

As a first step of the procedure, the measurements of flank wear VB and roughness parameter R_z we taken in order to determine the tool life curves $VB = f(t)$, determine tool live T values for critical tool wear, compute the average values of R_z roughness parameter for machining conditions selected for each experiment of the experimental design. In the next step, measurements of the cutting force components for the center point of the design and then measurements of the cutting force components for machining conditions selected for the other points of the design and for the given values of tool flank wear. These measurements were aimed at determining the relationships between cutting force components and tool wear measure as well as determining their average levels within the tool life period.

3. Mathematical model for the WT3-1 titanium turning process

An experimental model for the titanium turning process was built, based on typical program of polynomial regression and multiple stepwise regression. This model was used in the multi-criterion (multiple goal) optimization, allowing for the weights standing for significance of particular output quantities [6]. In this model, the following relationships are included: wear curve $VB = f(t/T)$, dimensionless relationships in accordance to formula 1 for roughness parameter R_z, for the cutting force components F_c F_f, F_n and the regression for T, R_z, Q_v, P_c, F_c, F_f, F_n, k_c versus cutting parameters.

The wear curve for dimensionless coordinate t/T (Fig. 1) was determined with typical polynomial regression analysis program, assuming the polynomial order $n = 9$, which has been defined by the formula:

$$VB = a_0 + \sum_{j=1}^{r} a_j \left(\frac{t}{T}\right)^r \tag{2}$$

It is obvious that at the moment $t = 0$, the tool wear equals 0, so that the a_0 constant factor value in the formula (2) should be zero. Still, the suggested value of a_0 is above zero, according to computation, since it is nearly equal to the value of the cutting edge radius $r_n = 0{,}02$. If the elastic-plastic displacement of the tool edge area is neglected, it can be assumed that for the time close to zero ($t > 0$), the tool edge abrasion on the tool flank takes place on the cylindrical part of the flank, height of which is equal to the tool cutting edge radius r_n. With this assumption, it can be stated that right after the beginning of the cutting process, the height of abrasion VB on the flank face is equal to the radius r_n. The values of the regression factors are listed in Table 2.

Table 2. Regression factor values for the tool edge wear vs. dimensionless t/T ratio

No.	Designation	Regression factor values
0	a_0	$0.0197362045070532 \approx r_n = 0.02$
1	a_1	4.12715707150360
2	a_2	-56.1673094875873
3	a_3	380.300325899130
4	a_4	-1425.15491882373
5	a_5	3173.05334143060
6	a_6	-4295.92449423669
7	a_7	3465.43205819143
8	a_8	-1528.85662637776
9	a_9	283.469166685179

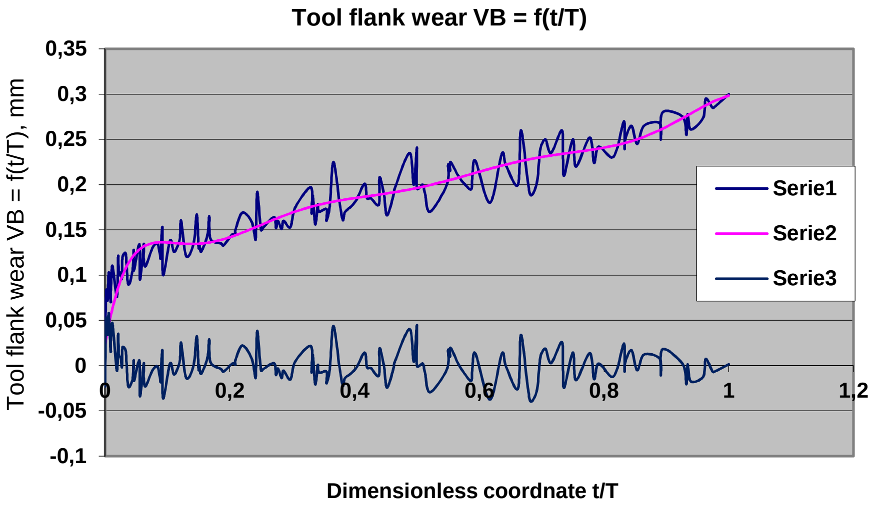

Fig. 1. Tool edge wear VB vs. dimensionless coordinate t/T. Serie 1 – measured points, Serie 2 – statistical model (approximation by a polynomial of $n = 9$ order), Serie 3 – regression residuals graph

Statistical assessment: coefficient of multiple correlation between *VB*, *t/T:* $R_{(VB,t/T)}$ = 0.974, Fisher ratio F = 316.248, number of measurements N = 163, number of the regression factors K = 9, critical F ratio for the assumed level of significance α = 0.05, for the number K of the numerator degrees of freedom and the number *N-K-1* of the denominator degrees of freedom (according to the F ratio definition) $F_{cr(\alpha,N\text{-}K\text{-}1,K)}$ = 1.95. The adequacy of the regression was examined by the F-test, which consists in determining the ratio which has been defined by the formula 3.

$$\frac{F_{(\alpha,N-K-1,K)}}{F_{kr(\alpha,N-K-1,K)}} \geq 1 \tag{3}$$

If the ratio $F/F_{kr} \geq 1$ then the regression is adequate which is the case in our calculations (F/F_{kr} = 162.23). According to formula 1, the dimensionless relationships were determined for the roughness parameter R_z, cutting force F_c, axial force F_f and radial force F_n (formulae 4÷7). The listed relationships take the following shape:

$$\frac{R_z}{R_{z,śr}} = 0,969 + 0,12\frac{t}{T} - 0,071\left(\frac{t}{T}\right)^2 \tag{4}$$

$$\frac{F_c}{F_{c,śr}} = 1,132 - 1,32\frac{t}{T} + 2,71\left(\frac{t}{T}\right)^2 - 1,47\left(\frac{t}{T}\right)^3 \tag{5}$$

$$\frac{F_f}{F_{f,śr}} = 0,864 - 0,056\frac{t}{T} + 0,528\left(\frac{t}{T}\right)^2 \tag{6}$$

$$\frac{F_n}{F_{n,śr}} = 0,908 + 0,042\frac{t}{T} + 0,230\left(\frac{t}{T}\right)^2 \tag{7}$$

The dimensionless relationship defined by the formula 4 is illustrated in Fig. 2.

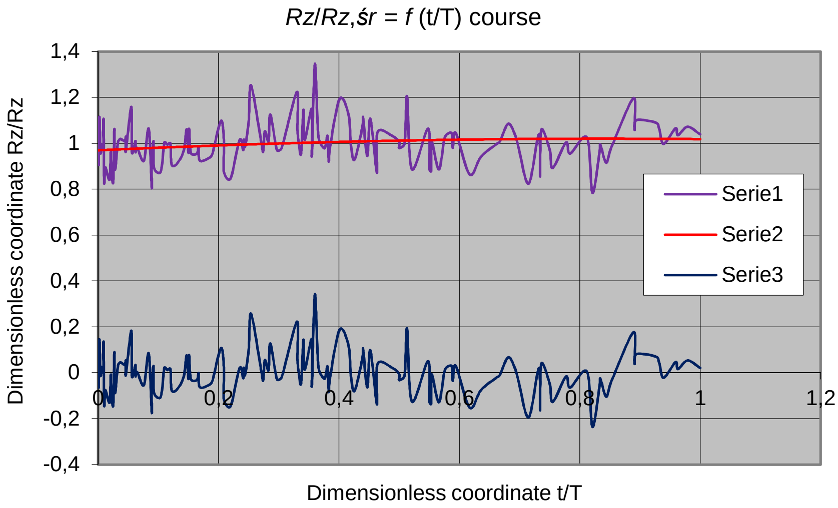

Fig. 2. Roughness parameters ratio $R_z/R_{z,śr}$ vs. dimensionless coordinate t/T. Serie 1 – measured points, Serie 2 – statistical model, Serie 3 – regression residuals graph

The mathematical model regression for describing WT3-1 titanium turning process is presented below (formulae 8÷15)

$$\begin{gathered} ln(T) = -27.634+24.486\, v_c - 4.794\, v_c^2 - 1.27\, f^2 - \\ - 1.384\, v_c f - 2.907\, v_c a_p - 4.151\, f a_p \end{gathered} \tag{8}$$

$$\begin{gathered} R_z = -0.945 + 2.731 a_p + 0.003\, v_c^2 + \\ + 185.271\, f^2 - 0.081 v_c\, a_p - 5.96352\, f a_p \end{gathered} \tag{9}$$

$$\begin{gathered} F_c = -171.96 + 16 v_c + 857.3 f + 82.44 a_p - 0.3441 v_c^2 - \\ - 3559.2 f^2 - 18.86578 a_p^2 + 1792.32 v_c a_p \end{gathered} \tag{10}$$

$$\begin{gathered} F_f = 127.82 - 5.11 v_c - 730.149 f - 73.0175 a_p + \\ + 29.1799\, v_c f + 2.9182 v_c a_p + 417.07516\, f a_p \end{gathered} \tag{11}$$

$$\begin{gathered} F_n = -69 + 599.54\, f + 344.11\, a_p + 0.1196\, v_c^2 - \\ - 82.0555\, a_p^2 - 3.1544\, v_c\, a_p + 187.081 f\, a_p \end{gathered} \tag{12}$$

$$\begin{gathered} k_c = 3962.773 + 62.8496\, v_c - 15672.671 f - 552.1 a_p - \\ - 1.35\, v_c^2 + 21351.75\, f^2 + 2001 f\, a_p \end{gathered} \tag{13}$$

$$P_c = 0.0393 - 0.104\, a_p - 0.0001 v_c^2 - 2.133\, f^2 + 0.0357\, v_c\, a_p + 0.00541 v_c\, f + 0.57348\, f\, a_p \tag{14}$$

$$Q_v = a_p\, f\, v_c \tag{15}$$

The depth-of-cut quantity a_p on Figures (3÷10) is subject to variation within 2.5 do 1.0 mm along the velocity axis.

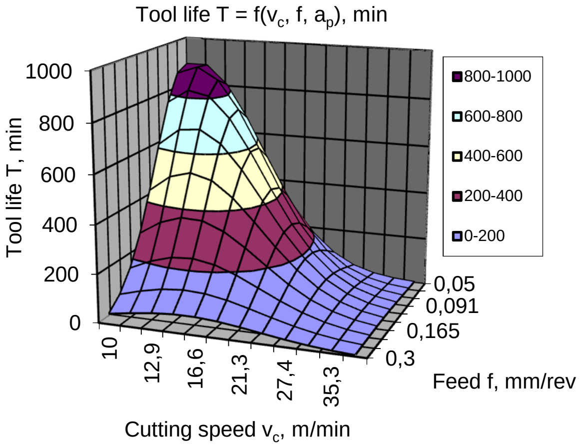

Fig. 3. Tool life relationship $T = f(v_c, f, a_p)$

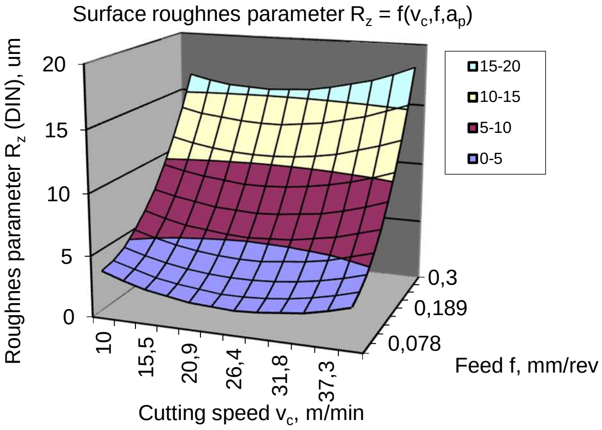

Fig. 4. R_z parameter relationship $R_z = f(v_c, f, a_p)$

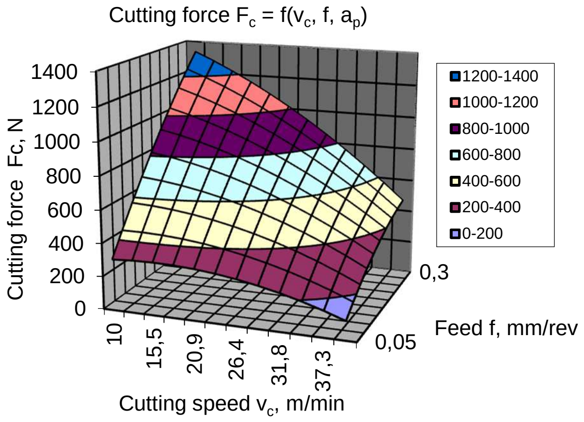

Fig. 5. Cutting force relationship $F_c = f(v_c, f, a_p)$

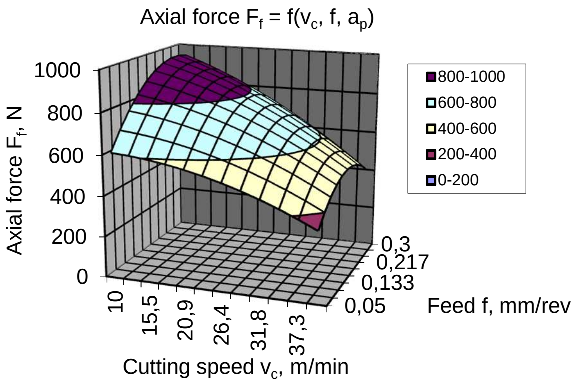

Fig. 6. Axial force relationship $F_f = f(v_c, f, a_p)$

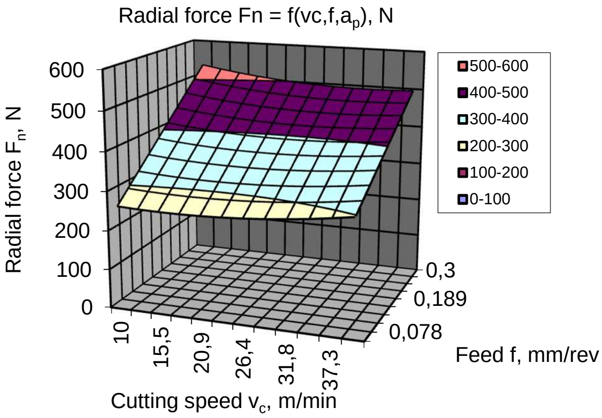

Fig. 7. Radial force relationship $F_n = f(v_c, f, a_p)$

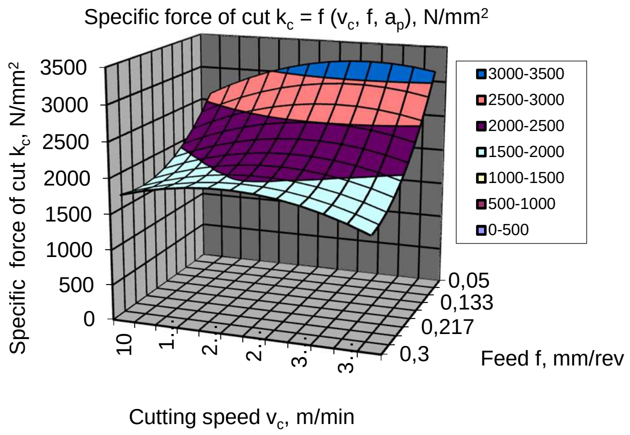

Fig. 8. Specific cutting force relationship graph $k_c = f(v_c f, a_p)$

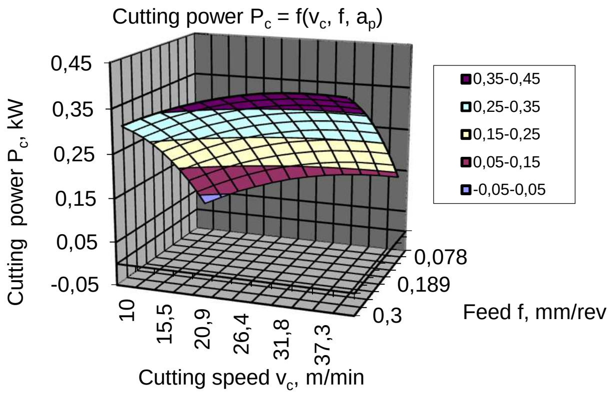

Fig. 9. Cutting power relationship graph $P_c = f(v_c, f, a_p)$

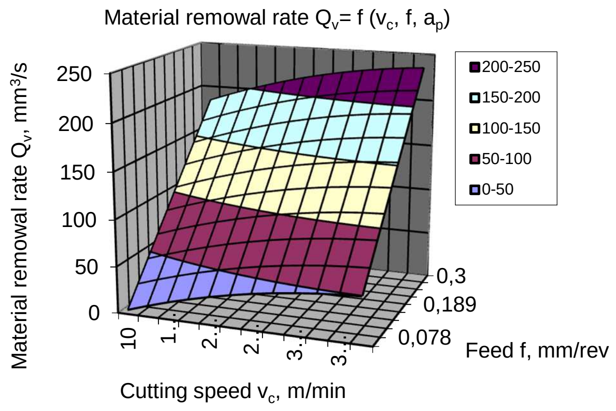

Fig. 10. Material removal rate relationship graph $Q_v = f(v_c, f, a_p)$

Adequacy of the regression was tested with the F-test at the significance level α = 0,05 (Table 3). Significance of the individual factors in the regression was test with the t-test at the same significance level.

Table 3. Assessment of the regression significance and of the regression factors significance

Regression number	**Degrees of freedom**		**Value of F-ratio**		$\frac{F}{F_{kr}}$	**Coefficient of correlation *R***	**Critical t_{kr} ratio at $\alpha = 0.05$**
	N-K-1	***K***	**computed**	**Critical $\alpha = 0.05$**			
8	13	6	16.677	2.955	5.644	0.945	2.160
9	14	5	221.457	2.960	78.816	0.994	2.145
10	12	7	348.521	2.910	119.77	0.998	2.179
11	13	6	451.435	2.955	152.77	0.999	2.160
12	13	6	36.395	2.955	12.152	0.972	2.160
13	13	6	20.530	2.955	6.947	0.950	2.160
14	13	6	213.035	2.955	72.093	0.995	2.160

4. Multi-criterion optimization

For technological applications, there are usually expectations connected with multi-criterion optimization with limitations which is called sometimes multiple-goal optimization or poly-optimization. Problems connected with this optimization type can be found in the available references [8, 9]. Practically, the dominant problem of multi-criterion optimization is replaced by a set of single criterion optimization problems. The adopted multi-criterion optimization usually includes the following the partial goals: the possibly highest productivity and the longest tool-life as well as the possibly lowest cutting power and surface roughness measured by the parameter R_z (DIN). The additional goal appears as determining the other performance features which have not been subjected to optimization, but remain technologically significant in the examined process. The limiting factors appear as some conditions precluding the use of force higher than permissible due to a number of factors like feed rate mechanism strength, interchangeable cutting insert strength, permissible deflection of the machined part, but also ensuring tool-life period T longer than 5 min. (according to the ISO standard). In order to determine the dominant optimization criterion, it is necessary to know the optimum values of the individual goal functions understood as minimum and maximum. It means that multi-criterion optimization should be preceded by single-criterion optimization for the individual functions of partial goals. For this purpose, custom optimization programs have been designed for single- and multi-criterion optimization employing a principle of systematic search. The results were presented in Table 4.

In order to determine the optimum set of machining parameters for the turning process, multi-criterion optimization based on correlation and weights and making use of dimensionless assessment of the OPT quantity as described by the following relationship:

$$OPT = \left\{ \sum_{i=1}^{n} W_i \frac{\alpha_i \cdot \left| Y_{\max,i} - Y_i \right|}{Y_{\max,i} - Y_{\min,i}} \right\}_{\max} \tag{16}$$

W_i – weight of each individual single-goal optimization criterion ($i = 1, 2, 3, \ldots n$), α – coefficient; $\alpha = +1$ – for a maximized quantity, $\alpha = -1$ – for a minimized quantity.

Table 4. Results of a single-goal optimization

Optimum values of the output quantities and corresponding machining parameters				
No.	**Maximum values**	v_c	f	a_p
1.	T = 834.585 min	14.848	0.103	2.227
2.	R_z = 18.082 μm	40.000	0.300	2.500
3.	F_c = 1391.937 N	22.727	0.093	2.500
4.	F_f = 999.619 N	25.455	0.260	2.500
5.	F_n = 509.856 N	10.000	0.300	1.818
6.	k_c = 3512.596 N/mm^2	23.333	0.050	1.000
7.	P_c = 0.535 kW	40.000	0.300	2.500
8.	Q = 500.000 mm^3/s	23.333	0.050	1.000
No.	**Minimum values**	v_c	f	a_p
1.	T = 268.334 MPa	25.455	0.267	2.424
2.	R_z = 68.210 μm	13.939	0.050	1.000
3.	F_c = 399.568 μm	39.697	0.055	1.000
4.	F_f = 1.122 mm	10.000	0.300	1.000
5.	F_n = 0.004 g/cm^2	22.727	0.050	2.500
6.	k_c = 0.146 %	40.000	0.300	1.000
7.	P_c = 9.639 μm	10.000	0.050	1.030
8.	Q_v = 8.333 mm^3/s	10.000	0.050	1.000

A value of the global assessment *OPT* may vary within the following limits:

$$0 < OPT < W_j \tag{17}$$

$$W_j = \sum_{i=1}^{n} W_i \tag{18}$$

The wanted optimum set of machining parameters v_c, f, a_p for the adopted variant of a single-goal fulfillment can be understood as such, for which the OPT expression value will be the highest. By assuming that all performance quantities of the process are of the same significance [10] which is equivalent to assigning them the weights $W_j = 1$, $i = 1, 2, 3, \dots, n$, the following set of optimum parameters was found and is presented in Table 5.

Table 5. Results of multi-criterion optimization for the adopted dominant criterion

Variant: $Q_{v\,max}$, $T_{\max}$, $P_{c\,min}$, $R_{z\,\min}$ ***Optimum parameters* (*$OPT_{\max}$*):** **v_c = 14.55 m/min. f = 0.053 mm/obr. a_p = 2.091**	
No.	**Optimum values of performance quantities**
1.	Qvx = 26.624 mm^3/s
2.	Tx = 831.837 min
3.	Pcx = 0.075 kW
4.	Rzx = 2.78 μm
5.	Fcx = 310.571 N
6.	Ffx = 588.207 N
7.	Fnx = 264.107 N
8.	kcx = 2892.430 N/mm^2
***Non optimum parameters* (*the worst – $OPT_{\min}$*):** **v_c = 23,03 m/min, f = 0, 3 mm/obr, a_p = 2,485**	
Lp.	**Non optimum values of performance quantities**
1.	Qvn = 286.134 mm^3/s
2.	Tn = 5.118 min
3.	Pcn = 0.531 kW
4.	Rzn= 14.99 μm
5.	Fcn= 980.136 N
6.	Ffn = 588.207 N
7.	Fnn = 480.487 N
8.	kcn = 2025.676 n/mm^2

Y_i – current value of the process performance parameter subjected to optimization, quantified with the given increment, $Y_{max,\,i}$ ($Y_{min,\,i}$) – maximum (minimum) value, n – number of the process parameters subjected to optimization

5. Conclusion

Typical solutions presented in the paper can be used for further research of the process, for improving the elaborated program of multi-criterion optimization with limitations and for applying the other types of multi-criterion optimization prepared by other known authors in order to find the best solution to problems occurring in the examined process.

The process of turning titanium and its alloys is difficult to examine, due to lack of stability. The developed mathematical model as well as optimization algorithms may become a foundation for the post-processing software in the CAM systems for automatic tool-path programming for the CNC machine tools.

References

[1] P. LASKOWSKI, W. HABRAT, K. KRUPA, J. SIENIAWSKI: Toczenie wykończeniowe stopu tytanu Ti-Al.-4V z zastosowaniem HCP. *Stal, Metale i Nowe Technologie*, (2013)11-12, 56-62.

[2] J. MAŁECKA: Stopy tytanu na osnowie faz międzymetalicznych TiAl(γ) i $Ti_3Al(\alpha_2)$. *Mechanik*, (2013)10, 888-890.

[3] K. KUBIAK: The influence of forging and hit treatment conditions on the microstructure and fatigue strength of die forgings made of two-phase titanium alloys Ti-6Al-4V and Ti-Al6-2Mo-2Cr. *Advances in Technology of Machines and Mechanical Equipment*, **21**(1997), 43-56.

[4] P. NIESŁONY, W. GRZESIK, W. HABRAT: Experimental and simulation investigations of face milling process of Ti-6Al-4V titanium alloy. *Advances in Manufacturing Science and Technology*, **39**(2015)1, 39-52.

[5] K.E. OCZOŚ: Kształtowanie ubytkowe tytanu i jego stopów w przemyśle lotniczym i technice medycznej, część I. *Mechanik*, **81**(2008)9, 888-890.

[6] K.E. OCZOŚ: Kształtowanie ubytkowe tytanu i jego stopów w przemyśle lotniczym i technice medycznej, część II. *Mechanik*, **81**(2008)10, 753-756.

[7] K. MAŃCZAK: Technika planowania eksperymentu. WNT, Warszawa 1976.

[8] W. POGORZELSKI: Teoria systemów i metody optymalizacji. OW PW, Warszawa 1996.

[9] M. KOWALCZYK: Application of Taguchi and Anova methods in selection of process parameters for surface roughness in precision turning of titanium. *Advances in Manufacturing Science and Technology*, **38**(2014)2, 21-35.

[10] W. TARNOWSKI: Modelowanie systemów. Wydawnictwo Uczelniane Politechniki Koszalińskiej, Koszalin 2004.

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