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ALGORITHMS FOR MEASUREMENT OF BENDED ELEMENTS BY MEANS OF MEASURING ARMS EQUIPPED WITH CONTACTLESS PROBES

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Summary

The paper presents proposals of algorithms for the control software for measuring arms equipped with the contactless probes (so called „V-probes”) as well as algorithms for bended elements' manufacturing-accuracy evaluation. The working principle of the V-probes is described. The critical analysis of the accuracy requirements specification for the bended elements is carried out. The typical measuring errors sources are pointed out.

Keywords: coordinate measuring technique, measuring arms, geometrical tolerances, contactless probe, tube inspection probe

Algorytm pomiaru elementów rurkowych za pomocą ramion pomiarowych z głowicą bezstykową

Streszczenie

W pracy przedstawiono opracowane algorytmy oprogramowania dla ramion pomiarowych wyposażonych w głowice bezstykowe V, także algorytmy prowadzenia analizy wyników pomiarów rurek powyginanych. Omówiono zasadę działania głowic bezstykowych V. Przeprowadzono krytyczną analizę dokładności wykonania elementów rurkowych. Wskazano typowe źródła błędów pomiaru.

Słowa kluczowe: współrzędnościowa technika pomiarowa, niepewność pomiaru, tolerancje geometryczne, głowice bezstykowe

1. Introduction

The optical measurements have many advantages comparing to conventional contact measurements [1]. The measuring arms used for the measurement of the bended elements (eg. pipe) can be equipped with hard probes, touch-trigger probes or non-contact (also called tube inspection probes or V-probes) [2-9]. The contactless probes (Fig. 1) for bended elements measurements are definitely most comfortable in use. Currently, the probes are

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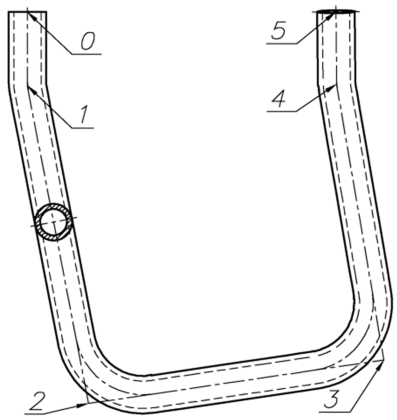
supplied with a specialized software (e.g. G-PAD [10]). The contactless probes are used for measurements of bended elements with different diameters and made of any material. For example Romer offers contactless probes in 6 sizes covering diameters of elements up to 130 mm [11]. The bended elements measurement results given by the software used with the measuring arms equipped with the V-probe consist of the lengths and the angles between the straight parts of the element. The software is also able to calculate the corrections for the bending machines parameters. Thanks to a coding system, the V-probes can be exchanged during the measurement without the need for probe requalification [10].

Fig. 1. The contactless probes [8, 9]

A standard way of dimensioning a bended element consists on giving the coordinates of the characteristic points (in the coordinate system of the final product e.g. car) and the bending radii (Fig. 2). In the industrial practice the manufacturing-accuracy-requirements specification for the bended element is still not enough precise which often leads to misunderstanding. Because the nominal shape of the bended element is specified by the characteristic points than a question arises: shall the tolerances be applied to the positions of those points?

The verification of such kind of parts is not standardized. Most often special check-gauges are used to verify the final product. Usually, such a check-gauge has a form of a cylindrical or rectangular groove the size which takes into account the tolerance for the bended element. This kind of verification is most strongly associated with profile any surface tolerance (Fig. 3) [12, 13]. Due to the fact that the elements are often not stiff (made of plastic) the check-gauges are designed to verify the bended element in the same orientation as in the final product. The element fulfils the requirement if it fits the check gauge, i.e. any part of it does not protrude over the edges of the check-gauge and the ends (mounting points) of the element are at certain positions marked with a tolerance on the gauge.

a)



b)

ID	Coordinates’		
	X	Y	Z
0	0,000	0,000	0,000
1	0,000	-43,309	0,000
2	35,931	-230,369	1,189
3	209,192	-204,843	6,924
4	180,963	-42,409	20,049
5	180,434	0,128	23,628

Fig. 2. Dimensioning of bended elements: a) simple example drawing, b) coordinates’ table

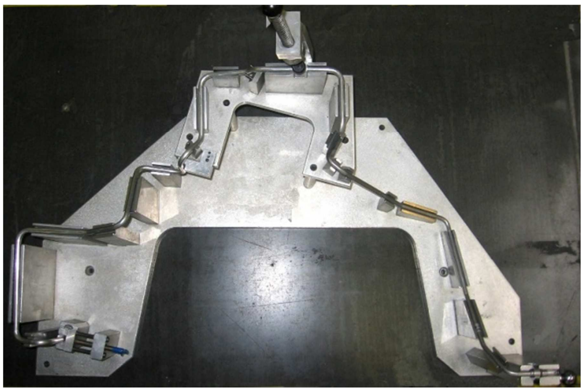


Fig. 3. Verification of bended elements; example check-gauge

In some cases, the check-gauge includes an element to which the bended element is mounted. It simulates the situation where one end of the element (with a connector) is a datum limiting 5 degrees of freedom.

The check-gauge enables verification if the product meets the requirements but does not give the values of the deviations which are necessary e.g. for SPC and/or process capability studies.

The paper presents a few algorithms for the software of measuring arms equipped with contactless probes and a few variants of measurement results evaluation for the bended elements. Basing on the research results the most important measurement error sources were identified.

2. The working principle of the V-probes

2.1. Mathematical model of measurement arm am equipped with V-probe

The V-probe is equipped with two laser systems consisting of laser beam emitter and the detector (Fig. 4). The design parameters of the V-probe are: the distance *S* of the beams intersection point from the last joint of the measuring arm and the angle α. The orientation of the probe is characterized by the vector of its axis (in the probe coordinate system $u = [0, 0, 1]$) and the normal vector of the beams plane (in the probe coordinate system $v = [1, 0, 0]$). The directional vectors for particular beams in the probe coordinate system are: $w_1 = [0, \cos\alpha, \sin\alpha]$, $w_2 = [0, \cos\alpha, -\sin\alpha]$.

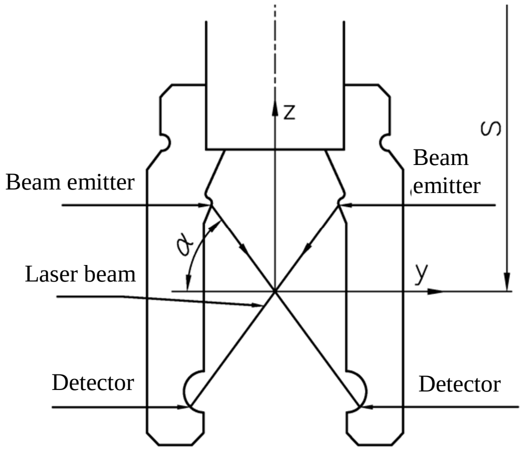


Fig. 1. Scheme of the V-probe with the design parameters *S* and α

The proper model enabling calculation of the position and orientation of the probe during the probing in the measuring arm coordinate system is based on the forward kinematics model (Fig. 5) using Denavit-Hartenberg convention (Eq. 1) [14-17].

A single joint’s pose matrix takes form:

$$A_i = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & l_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & l_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & k_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

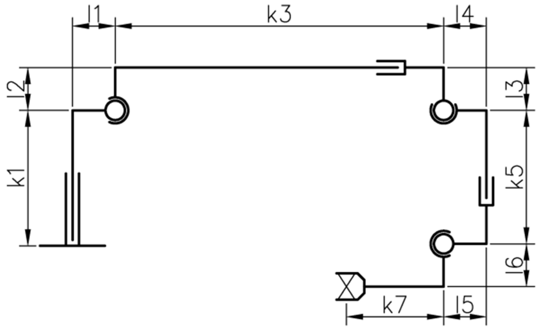


Fig. 5. Kinematic model of measuring arm used for the measuring errors analysis of bended elements

The V-probe position and orientation is given in the D-H model matrix **T** (required for the simulation) resulting from the joints' matrixes [18-20]:

$$\mathbf{T} = \mathbf{A}_1 \cdot \ldots \cdot \mathbf{A}_n \tag{2}$$

The coordinates of the probe's characteristic point (beams' intersection point for the V-probe) are given in the elements $\mathbf{T}_{14}$, $\mathbf{T}_{24}$, $\mathbf{T}_{34}$, and the orientation of the probe (the axes' direction vectors) are given in the following elements: x axis vector – ($\mathbf{T}_{11}$, $\mathbf{T}_{21}$, $\mathbf{T}_{31}$), y axis vector – ($\mathbf{T}_{12}$, $\mathbf{T}_{22}$, $\mathbf{T}_{32}$), z axis vector – ($\mathbf{T}_{13}$, $\mathbf{T}_{23}$, $\mathbf{T}_{33}$).

The V-Probe generates the measuring signals when whichever of the laser beams is interrupted and also when a beam reconnects the emitter and the detector. In the same moment the software records the position and orientation of the probe as well as the number of the beam which generated the signal.

2.2. The measurement algorithms

The used algorithms are not published but it is obvious that during the movement of the probe in relation to the measured element (both towards the element and back) the following situations consecutively take place (Fig. 6): both detectors are lit – one detector is lit – none of the detectors is lit – one detector is lit – both detectors are lit (the return move starts here) – one detector is lit – none of the detectors is lit – one detector is lit and again both detectors are lit.

During the measurement of the particular bended element axis points the user is asked to align the probe perpendicularly to the axis of the measured element and to lead the probe along a straight line and not to change its orientation (Fig. 6a). Additionally, to determine the position of the element end points it is necessary to move the probe to the element end while preserving its proper orientation in regard to the element axis (the beams plane perpendicular to the element axis) (Fig. 6b). The precise fulfilling those requirements is of course impossible. Further, the influence of that on the measurement uncertainty will be discussed.

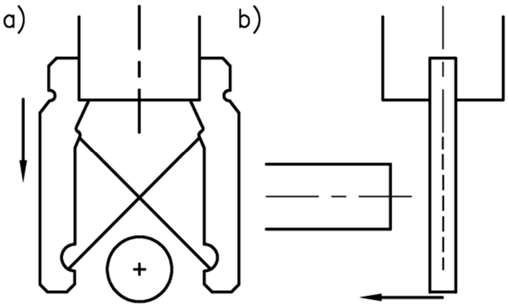


Fig. 6. Relative position and orientation of the probe and the bended element during: a) measurement of the axis point, b) determination of the element endpoint

Algorithm 1. Calculation of the coordinates of one axis point using minimum amount of data. If we assume that the V-probe can be moved exactly along straight line in plane perpendicular to the axis of the element and that the element diameter is known (the diameter of the measured element is input by the user prior the measurement) than to determine the coordinates of a **point of the element axis** it is enough to move the probe until sequentially both beams are disturbed. The axis point coordinates are calculated from the intersection of the two straight lines parallel to the laser beams and shifted in the beams plane (in appropriate direction) by the value of the element radius (Fig. 7a). It can be proved that a possible rotation (during the measurement) of the probe around the element axis does not introduce any error (Fig. 7b). The incorrect value of the diameter, or moving the probe without maintaining the perpendicular orientation of the beams plane in regard to the element axis, results the measured element position error in the probing direction.

Taking into account the fact that the V-probe is not moved in one plane, the mentioned straight lines also are not lying in the same plane, therefore to calculate the axis point it is necessary to calculate a projection plane (an approximation of the plane perpendicular to the element's axis) on which the axis point is to be calculated from the projected straight lines.

The accuracy of the measurement with the V-probe is influenced, among others, by the real value of the measured element diameter (which is inputted by the user) and the real orientation of the probe (difference to perpendicular) in relation to the element axis (Fig. 8).

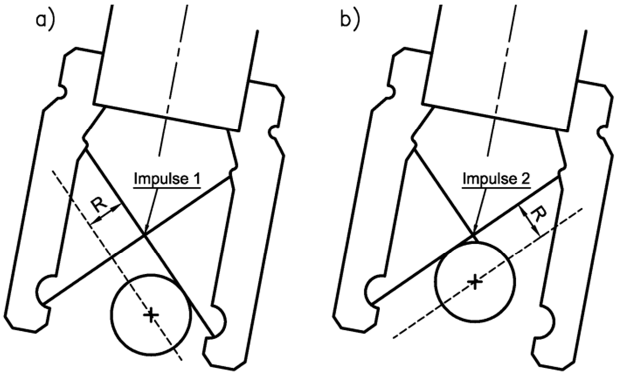


Fig. 7. The algorithm of determination of element's axis point on the basis of the known element radius *R* and the measuring signals generated by interrupting the 2 laser beams: a) interruption of the first beam, b) interruption of the second beam

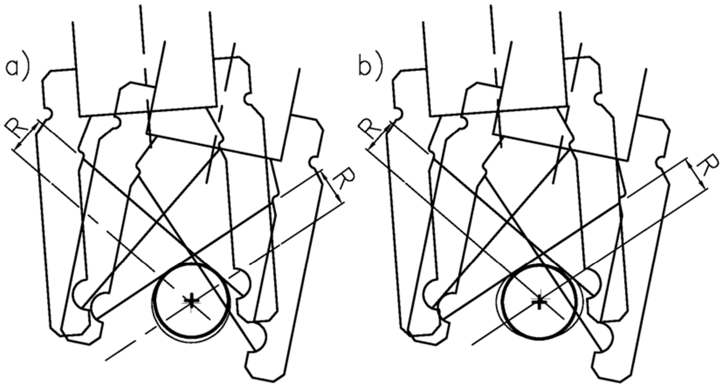


Fig. 8. Explanation of the measurement error sources: a) incorrect value of the element diameter, b) tilt of the V-probe plane in relation to the element axis

If you take a close looks at the Fig. 8 it may seem that if the real diameter of the element is different from the value input to the software or the V-probe plate is not perpendicular to the element's axis than the measured point of the axis will be shifted in relation to the real axis point.

Algorithm 2. Possibility of compensating the errors which are characteristic for the algorithm 1. In the full movement sequence in one direction the axis point can be calculated twice and its coordinates can be calculated as an average of the two recorded points. In the complete sequence "there-and-back" the axis point can be calculated 4 times and as the final result the average from 4 results is taken what additionally improves the accuracy.

Algorithm 3. Calculating the parameters of the straight fragment of the bended element. Using the algorithm 1 or 2 the straight fragment can be calculated in the next step as a straight line constructed from two or more points. There is a possibility to determine the straight fragment in one step by solving a 3D task.

Due to the fact that the measurements are carried out in 2 sections one gets, respectively to the chosen precision, eight or sixteen lines tangential to the element with given diameter. The determination of the axis of a straight fragment of a bended element involves solving of the following mathematical problem: find a straight line which is apart from each of the 8 or 16 straight lines by approximately the same distance (radius of the element).

The distance d between a straight line k defined by the point K and the vector u and the straight line l defined by a point L and a vector v can be expressed by following formula:

$$d = \left| \frac{(K - L) \cdot (u \times v)}{|u \times v|} \right| \tag{3}$$

Unknown parameters L and v of the straight line l representing the straight segment of the bended element axis can be calculated by minimising the sum of squares of distances of 8 (or 16) straight lines k_i from the line l (Fig. 9):

$$\sum_{i=1}^{8(16)} \left(\frac{(K_i - L) \cdot (u_i \times v)}{|u_i \times v|} \right)^2 \rightarrow \min \tag{4}$$

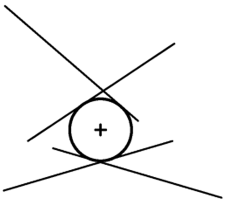


Fig. 9. Illustration for the formulae (3 and 4). View of the measurement results (tangential straight lines) for one measuring section in case of “single precision” measuring mode

Thanks to the known parameters of the V-probe (the beams angle and the normal vector of the beams plane), the straight lines k_i can be defined by indications of the articulated arm encoders and the V-probe characteristic points K_i (the intersection of the beams) and the V-probe definition vectors (the directions of the beams) in the moment of generating the measuring signals.

3. Calculating the profile any surface deviation

The following algorithm (incorrect in the authors’ opinion) is most often used for measurement of bended elements. After determining the straight segments of the bended element axis the intersection points of particular segments are calculated. Because the real axes of the straight segments do not intersect (do not lie in the same plane) the software algorithms as the result of the intersection usually give the middle point of the shortest line connecting the axes.

In order to determine the end-points of the measured element it is necessary move the V-probe towards the end-faces of the element (along the element’s axis) in order the beams intersect the end-face of the bended element.

Authors propose the following algorithm (Fig. 10):

• measure the straight segments axes according to the algorithm 3 described above,

• calculate the points A_{ij} (i = 1 ... 2; j – depending on the number of the straight segments) of axis near the ends of the straight segments of the bended element (near the probing sections),

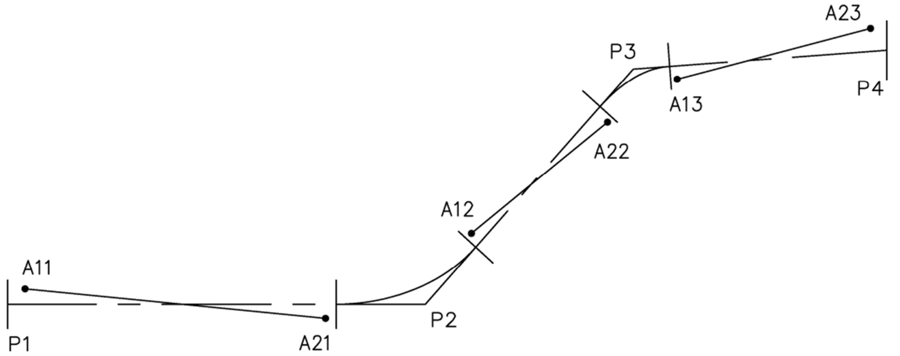


Fig. 10. The graphical explanation of the deviation calculation algorithm

- find a coordinate-system-transformation matrix resulting from the best fit of the point set A_{ij} to the nominal axis of the bended element defined by the characteristic points, wherein the fitting criterion shall be sum of squares of distances between the points A_{ij} and the straight lines defined by the neighbouring characteristic points P_j and P_{j+1}:

$$\sum_{j=1}^{3}\sum_{i=1}^{2}\frac{a_{ij}^2+b_{ij}^2+c_{ij}^2}{l_j^2}\rightarrow \min \tag{5}$$

$$\begin{aligned}
a_{ij} &= \left(y_{Pj}-y_{Aij}\right)\left(z_{P(j+1)}-z_{Pj}\right)-\left(z_{Pj}-z_{Aij}\right)\left(y_{P(j+1)}-y_{Pj}\right)\\
b_{ij} &= \left(z_{Pj}-z_{Aij}\right)\left(x_{P(j+1)}-x_{Pj}\right)-\left(x_{Pj}-x_{Aij}\right)\left(z_{P(j+1)}-z_{Pj}\right)\\
c_{ij} &= \left(x_{Pj}-x_{Aij}\right)\left(y_{P(j+1)}-y_{Pj}\right)-\left(y_{Pj}-y_{Aij}\right)\left(x_{P(j+1)}-x_{Pj}\right)
\end{aligned} \tag{6}$$

$$l_j^2=\left(x_{P(j+1)}-x_{Pj}\right)^2+\left(y_{P(j+1)}-y_{Pj}\right)^2+\left(z_{P(j+1)}-z_{Pj}\right)^2 \tag{7}$$

- calculate distances d_{ij} of all calculated axes' points and the nominal axis:

$$d_{ij}=\frac{\sqrt{a_{ij}^2+b_{ij}^2+c_{ij}^2}}{l_j} \tag{8}$$

- the profile any surface deviation can be calculated as the doubled biggest distance d_{ij}

$$\Delta = 2\cdot \max\left(d_{ij}\right) \tag{9}$$

- determine the coordinates of the end points and project them onto the nominal axis,
- as the deviation of position of the end points (to be treated roughly as position of the end-faces) assume doubled value of the distance of these points to their respective nominal position.

4. Concluding remarks

The algorithms used in the software of the articulated measuring arms equipped with contactless tube inspection probes (V-probes) were presented. Also a proposal of the algorithm for best-fit and manufacturing accuracy of the bended tubes was presented for the case of profile any surface tolerance without a datum system. The algorithms are not published by the producers but awareness of them is necessary for assuming proper measuring strategy and proper interpretation of the results.

The accuracy testing of measuring arms equipped with V-probe needs further development e.g. using Monte Carlo method. Probes of this type are used more and more often especially for the elements made of soft materials (e.g. plastics), measurement of which with the use of contact probes is sometimes impossible.

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