ESTIMATION OF NONLINEAR MODELS' PARAMETERS OF MACHINE TOOL SUPPORTING SYSTEMS USING INCOMPLETE VIBRATION TESTS DATA

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S u m m a r y

An algorithm of reconstruction nonlinear motion equations of machine tool supporting system dynamic models, on the basis of incomplete information about an object is presented in this paper. This information is obtained from experimental tests in the form of time response characteristics of an object to a given excitation. Tested models are developed in the rigid finite elements method convention supplemented by slideway joint modeling option. The effectiveness of the algorithm has been verified by means of simulated identification of selected parameters of the knee – column slideway joint nonlinear model of universal milling machine.

Keywords: identification, incomplete information, nonlinear models, machine tools

Estymacja parametrów nieliniowych modeli układów nośnych obrabiarek na podstawie niepełnej informacji z badań doświadczalnych

S t r e s z c z e n i e

W pracy przedstawiono algorytm rekonstrukcji nieliniowych równań ruchu modeli układów nośnych obrabiarek na podstawie niepełnej informacji o obiekcie uzyskiwanej z badań doświadczalnych w postaci przebiegów czasowych odpowiedzi obiektu na zadane wymuszenie. Badane modele tworzone są w konwencji metody sztywnych elementów skończonych uzupełnionej o opcję modelowania prowadnic. Efektywność algorytmu zweryfikowano poprzez symulowaną identyfikację wybranych parametrów nieliniowego modelu połączenia prowadnicowego wspornik–korpus frezarki uniwersalnej.

Słowa kluczowe: identyfikacja, niepełna informacja, nieliniowe modele, obrabiarki

1. Introduction

The continuous and natural trends towards improvement of dynamic properties of machine tools with parallel pressure on heightening the quality and

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efficiency of machining, forced constructors to design structures much more resistant to excitation and propagation of vibrations. This in turn compelled creation and development of currently used methods and techniques of shaping and improving dynamic properties of machine tools, starting from the phase of creation of a new design of the machine, through the phase of building its prototype up till the modification of already existing machines.

Effective performance of these tasks would not be possible without using techniques of assisting structural design, and particularly without computer simulation tests, which radically reduce, to indispensible minimum, and in many cases completely eliminate the need of carrying out expensive, time consuming and very often dangerous experiments. However, conducting simulation tests requires an earlier creation of reliable and adequate physical and mathematical models of the tested machines.

An effective tool for developing linear models of multibody structures was Rigid Finite Element (RFE) method elaborated by Kruszewski et al. [1, 2]. In relation to machine tools supporting systems this method was complemented by Szwengier's option [3] which makes it possible to effectively model slideway joints. Nevertheless, in the case of large displacements of particular subassemblies of machine tools and in the case of small cutting forces, which appear in nowadays commonly used high speed machining, the use of linear models turns out to be a too large and too unjustified simplification, which makes it impossible to correctly predict dynamic properties of machine tools. It is connected with nonlinear characteristics of contact joints in a machine tool slideway system.

For this reason the authors of this paper elaborated two algorithms of reconstructing nonlinear equations of motion and used them to identify nonlinear models of machine tool supporting systems. The models were developed in the RFE method convention. Identification was carried out on the basis of experimentally determined time response characteristics of the object to a given force excitation. The first of these algorithms was described in the authors' previous paper [4] and it was designated to identify parameters of the above mentioned models on the basis of complete information of an object from experimental tests.

This paper presents the second algorithm. It was elaborated for the case when it is impossible to obtain, from experimental tests, full information of the object, i.e. if there is no such possibility or it is pointless, e.g. because of large costs, to determine the full time runs of response (displacements, velocities and accelerations) of a real object to a given excitation, measured in rigorously defined directions and rigorously defined points.

2. Nonlinear models of machine tools

In both algorithms the parameters of spring-damping elements of machine tool supporting systems models, created in RFE method convention, can be estimated. The way of developing such models is strictly described in paper [4]. For that reason in this work, to avoid replacements, only the main dependencies are given.

The machine tool is modeled as a spatial system of rigid finite elements (RFEs) interconnected with spring-damping elements (SDEs). The SDEs that describe slideway joints, turn tables and rolling guides consist of six weightless springs which model three translational and three rotational stiffnesses and dampings. Leading screws are modeled using one weightless spring located along the axis of the screw, whereas SDEs that model vibration isolation pads consist of three such springs.

Particular SDEs can have linear or nonlinear characteristics. The SDEs that model slideway joints were assumed to have nonlinear stiffnesses and linear dampings. Other SDEs were assumed as linear. The motion equation of the model has the following form:

$$\boldsymbol{M}\cdot\ddot{\boldsymbol{q}}+\boldsymbol{C}\cdot\dot{\boldsymbol{q}}+\boldsymbol{K}\cdot\boldsymbol{q}+\boldsymbol{F}_d(\dot{\boldsymbol{q}})+\boldsymbol{F}_e(\tilde{\boldsymbol{q}})=\boldsymbol{P}(t) \tag{1}$$

where: $\boldsymbol{M}$ – diag$\{m_1, m_1, m_1, J_{11}, J_{21}, J_{31}, ..., m_u, m_u, m_u, J_{1u}, J_{2u}, J_{3u}\}$ – inertia matrix, $\boldsymbol{C}$, $\boldsymbol{K}$ – damping and stiffness matrices of SDEs that model leading screws, turn tables, rolling guides and vibration isolation pads, $\boldsymbol{q}$ – col$\{q_{11}, q_{21}, ..., q_{61}, ..., q_{1u}, q_{2u}, ..., q_{6u}\}$ – vector of generalized coordinates, $\boldsymbol{F}_e(\tilde{\boldsymbol{q}})$ – vector of nonlinear restoring forces modeling elastic interaction of mating surfaces of slideway joints, $\tilde{\boldsymbol{q}}$ – col$\{q_{1i}^m, q_{2i}^m, ..., q_{6i}^m\}$ – vector with nonlinear components, $\boldsymbol{F}_d(\dot{\boldsymbol{q}})$ – vector of linear damping forces in the slideway joints, $\boldsymbol{P}(t)$ – vector of generalized excitation forces, m – exponent, parameter determined experimentally, characterizes nonlinear properties of slideway joint (usually $m = 1\div3$), u – number of rigid finite elements (RFEs) in a model.

Nonlinear generalized restoring forces $\boldsymbol{F}_{e_{r\kappa}}$ and $\boldsymbol{F}_{e_{p\kappa}}$ modeling elastic interaction of mating surfaces of *r-th* and *p-th* RFEs which describe deformations $\boldsymbol{\delta}_\kappa$ of *κ-th* SDE have the same value but opposite sense. They can be expressed as [4]:

$$\boldsymbol{F}_{e_{r\kappa}}=\boldsymbol{D}_{r\kappa}\cdot\boldsymbol{a}_\kappa \quad \text{and} \quad \boldsymbol{F}_{e_{p\kappa}}=-\boldsymbol{D}_{p\kappa}\cdot\boldsymbol{a}_\kappa \tag{2}$$

where: $\boldsymbol{a}_\kappa$– matrix of stiffness coefficients of the SDE of a number κ, which depend on the surface geometry of the slideway joint that is modeled by this SDE.

This matrix has the form:

$$\boldsymbol{a}_\kappa = [a_{\kappa 1}, a_{\kappa 2}, ..., a_{\kappa 6}]^T \tag{3}$$

The way of calculation values of particular elements $a_{\kappa i}$ of vector $\boldsymbol{a}_\kappa$, assuming rectangular load distribution on the active part of the given slideway is described in the paper [4].

Matrices $\boldsymbol{D}_{r\kappa}$ and $\boldsymbol{D}_{p\kappa}$ are defined as:

$$\left.\begin{aligned} \boldsymbol{D}_{r\kappa} &= \boldsymbol{S}_{r\kappa}^T \cdot \boldsymbol{\theta}_{r\kappa}^T \cdot diag\{\tilde{\boldsymbol{\delta}}_\kappa\} \\ \boldsymbol{D}_{p\kappa} &= -\boldsymbol{S}_{p\kappa}^T \cdot \boldsymbol{\theta}_{p\kappa}^T \cdot diag\{\tilde{\boldsymbol{\delta}}_\kappa\} \end{aligned}\right\} \tag{4}$$

where: $\boldsymbol{\theta}_{r\kappa}$ and $\boldsymbol{\theta}_{p\kappa}$ – matrices of direction cosines of angles between principal axes system of κ-*th* SDE and principal central axes of inertia systems of RFEs *r* and *p*, respectively, $\boldsymbol{S}_{r\kappa}$ and $\boldsymbol{S}_{p\kappa}$ – matrices of attachment for a SDE with number κ to RFEs *r* and *p*, respectively, $\tilde{\boldsymbol{\delta}}_\kappa$ – vector whose components are nonlinear function of deformation $\boldsymbol{\delta}_\kappa$ of the SDE of a number κ

where:

$$\boldsymbol{\delta}_\kappa = \Delta \boldsymbol{w}_{rp\kappa} = \boldsymbol{w}_{r\kappa} - \boldsymbol{w}_{p\kappa} \tag{5}$$

and $\boldsymbol{w}_{r\kappa}$ – displacements vector of the end of SDE of a number κ attached to the *r-th* RFE, $\boldsymbol{w}_{p\kappa}$ – displacements vector of the end of SDE of a number κ attached to the *p-th* RFE,

$$\tilde{\boldsymbol{\delta}}_\kappa = \mathrm{col}\{\delta_{1\kappa}^m, \delta_{2\kappa}^m, ..., \delta_{6\kappa}^m\} \tag{6}$$

Generalized dissipative forces vector, in turn, has the form [4]:

$$\boldsymbol{F}_{d_{r\kappa}} = \boldsymbol{E}_{r\kappa} \cdot \boldsymbol{h}_\kappa \quad \text{and} \quad \boldsymbol{F}_{d_{p\kappa}} = -\boldsymbol{E}_{p\kappa} \cdot \boldsymbol{h}_\kappa \tag{7}$$

where:

$$\left.\begin{aligned} \boldsymbol{E}_{r\kappa} &= \boldsymbol{S}_{r\kappa}^{T} \cdot \boldsymbol{\theta}_{r\kappa}^{T} \cdot diag\{\dot{\boldsymbol{\delta}}_{\kappa}\} \\ \boldsymbol{E}_{p\kappa} &= -\boldsymbol{S}_{p\kappa}^{T} \cdot \boldsymbol{\theta}_{p\kappa}^{T} \cdot diag\{\dot{\boldsymbol{\delta}}_{\kappa}\} \end{aligned}\right\} \tag{8}$$

and

$$\boldsymbol{h}_{\kappa} = [h_{\kappa 1}, h_{\kappa 2}, ..., h_{\kappa 6}]^{T} \tag{9}$$

The way of calculation values of particular elements $h_{\kappa i}$ of damping vector $\boldsymbol{h}_{\kappa}$ is described in paper [4].

In the elaborated algorithm it is assumed that inertia parameters (masses and moments of inertia) of the model, which is described by equation (1), are known. It is assumed also, that values of some parameters which describe spring-dissipative properties of this model are known too, e.g. values of stiffness and damping coefficients of vibration isolation pads on which the machine tool is founded or values of stiffness and damping coefficients of leading screws. The known data are grouped in inertia, stiffness and damping matrices $\boldsymbol{M}$, $\boldsymbol{K}$ and $\boldsymbol{C}$, respectively.

The unknown parameters of the model are stiffness and damping coefficients of slideway joints. It is assumed that the damping forces $\boldsymbol{F}_d$ are linearly dependent on the velocity, while restoring forces $\boldsymbol{F}_e$ can be both nonlinear and in particular case linear.

Identified are the damping coefficients $h_{\kappa i}$ that are used for determination of the damping forces $\boldsymbol{F}_d$ in slideway joints, and coefficients $a_{\kappa i}$, that at known exponent m univocally define restoring forces $\boldsymbol{F}_e$ of joints.

3. Algorithm of the identification

In this paper an algorithm that uses incomplete information of modeled machine tool is presented. This information is obtained from experimental tests. The purpose of the identification is to find estimates of the unknown parameters that characterize nonlinear stiffness and linear damping of particular contacts of slideway joints.

The dependences between response signals $\boldsymbol{y}(t)$ of tested object determined experimentally and parameters $\boldsymbol{x}$ of the model, assumed for the identification, are nonlinear and they are burdened with systematic and random errors. After assumption that systematic errors can be eliminated when signal processing, these characteristics, in any case of estimation of a nonlinear time-invariant system, can be described in the following general form (Schweppe [5]):

$$\boldsymbol{y}(t) = \boldsymbol{g}(t, x) + \boldsymbol{\xi}(t) \tag{10}$$

where: $\boldsymbol{y}(t)$ – the measured characteristic of the object, $\boldsymbol{g}(t,x)$ – the same characteristic calculated for the model, $\boldsymbol{\xi}(t)$ – multi component observational noise, x – vector of model parameters that are to be identified

$$\boldsymbol{x} = [x_1, x_2, ..., x_r]^T \tag{11}$$

where: r – number of decision variables (parameters to be identified).

As it was said in the presented algorithm the identification of model parameters is carried out on the basis of incomplete information of the object. In adopted Fisher's model of uncertainty it is assumed that error $\boldsymbol{\xi}$ is a random vector with a known function of probability density $p(\boldsymbol{\xi})$, and the estimated vector of parameters $\boldsymbol{x}$ is completely unknown. At the assumption that vector $\boldsymbol{\xi}$ has a normal distribution $N(\mathbf{0}, \boldsymbol{R})$, the function of probability distribution $\boldsymbol{y}$ for a given $\boldsymbol{x}$, i.e. the likelihood function $p(\boldsymbol{y}: \boldsymbol{x})$ has the form:

$$p(\mathbf{y} : \mathbf{x}) = [(2\pi)^{\mathrm{N}} |\mathbf{R}|]^{-1/2} \exp\{-\tfrac{1}{2}[\mathbf{y} - \mathbf{g}(\mathbf{x})]^{\mathrm{T}} \cdot \mathbf{R}^{-1} \cdot [\mathbf{y} - \mathbf{g}(\mathbf{x})]\} \tag{12}$$

The strongest estimator for Fisher's models is estimator of maximum likelihood, i.e. one which maximizes the function $p(\boldsymbol{y}: \boldsymbol{x})$ for $\boldsymbol{y} = \boldsymbol{y}_{real}$. In the analyzed case, this is equal to minimization of the relationship:

$$J(\boldsymbol{x}) = [\boldsymbol{y} - \boldsymbol{g}(\boldsymbol{x})]^{\mathrm{T}} \cdot \boldsymbol{R}^{-1} \cdot [\boldsymbol{y} - \boldsymbol{g}(\boldsymbol{x})] \tag{13}$$

At the assumption that variances of errors $\boldsymbol{\xi}_i$ are the same in particular moments of time, it can be written:

$$\boldsymbol{R}^{-1} = \frac{1}{\sigma_{\xi}^2} \cdot \boldsymbol{I} \tag{14}$$

where: $\boldsymbol{I}$ – identity matrix.

At such assumption, and in the case of carrying identification on the basis of incomplete information of an object, when measured vector is formed only by accelerations, i.e. when:

$$\boldsymbol{y}(t) = \ddot{\boldsymbol{z}}(t) \quad \text{and} \quad \boldsymbol{g}(t, \boldsymbol{x}) = \ddot{\boldsymbol{q}}(t, \boldsymbol{x}) \tag{15}$$

the criterion given by relationship (13) is reduced to the form:

$$J(\boldsymbol{x}) = [\ddot{\boldsymbol{z}} - \ddot{\boldsymbol{q}}(\boldsymbol{x})]^T \cdot [\ddot{\boldsymbol{z}} - \ddot{\boldsymbol{q}}(\boldsymbol{x})] \tag{16}$$

Vectors $\ddot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{z}}$ contain acceleration signals of time realizations selected for the evaluation of the model and object consistency. Since $\ddot{\boldsymbol{q}}$ is a nonlinear and generally unknown function of the estimated parameters $\boldsymbol{x}$ this problem can only be solved using iterative methods of optimization.

Efficiency of these methods measured as the number of iteration required for finding the global minimum depends on the selection of a starting point $\boldsymbol{x}$ and its distance from optimal solution. For nonlinear models finding the global minimum is a particularly complex task. A relatively simple solution is provided by constraining decision variables to the so-called variability cubic on the basis of literature or previous experimental data:

$$x_i^{min} \leq x_i \leq x_i^{max}, \qquad i = 1, 2, ..., r \tag{17}$$

and then the r dimensional parameters space constructed in such a way is systematically searched for the global minimum.

Such a search is performed according to the planned numerical experiment which consists in the generation of N_p uniformly distributed random numbers which form experiment design and then calculation of the objective function J at each of these points in order to find the best one. This approach, however, is characterized by a non-uniform location of points in the parameters space for a small number of these points.

The presented algorithm of identification employs highly uniform plans that are called in literature as LP_τ plans. A procedure of creating these plans is given by Sobol and Levitan, [6]. The determination of coordinates of LP_t plan points is as follows: for a given point number $i = 1,\ldots, N_p$ we first calculate the number w:

$$w = 1 + [\ln i / \ln 2] \tag{18}$$

and then for $j = 1, 2, ..., r$ we calculate:

$$xp_{ij} = \sum_{k=1}^{w} 2^{-k+1} \cdot \left\{ \frac{1}{2} \sum_{l=k}^{w} [2 \cdot \{i \cdot 2^{-l}\}] \cdot [2 \cdot \{S_j^{(l)} \cdot 2^{k-l-1}\}] \right\} \tag{19}$$

where: xp_{ij} – determines the normalized value of the j-th decision variable in the i-th point of the plan.

In equations (18) and (19) it is taken that the notation $[z]$ indicates integer and $\{z\}$ – fractional portion of z whereas $S_j^{(l)}$ are constants that can be found in a table presented in the above mentioned paper by Sobol and Levitan. The great advantage of the plans based on LP_τ series is that extending the number of points from N_p to N_{p1} does not require repeating calculations for the first N_p points.

Owing to its good efficiency in finding global minimum the Broyden-Fletcher-Goldfarb-Shanno (B-F-G-S) algorithm of variable metric (Papalambros and Wilde [7]) has been implemented. In this algorithm the recurrent relation takes the following form:

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k - \lambda_k \cdot \boldsymbol{d}_k \tag{20}$$

where: λ_k – step size that minimizes objective function along the search direction $\boldsymbol{d}_k$

$$\boldsymbol{d}_k = \boldsymbol{B}_k \cdot \nabla J(\boldsymbol{x}_k) \tag{21}$$

$$\boldsymbol{B}_k \approx \boldsymbol{H}_k^{-1} \tag{22}$$

$\nabla J(\boldsymbol{x}_k)$ – gradient of the objective function at the point $\boldsymbol{x}_k$, $\boldsymbol{H}_k$ – Hessian of the objective function at the point $\boldsymbol{x}_k$.

The value of the gradient $\nabla J(\boldsymbol{x}_k)$ of the objective function J can be accurately approximated by differential quotients. Gradient estimation of this function at the point $\boldsymbol{x}_k$ consists in calculating its value at few points $\boldsymbol{x}_k^j$ at the vicinity of $\boldsymbol{x}_k$. The values of gradient's estimates are computed from the differences of the objective function at those points.

Matrix $\boldsymbol{B}_k$ approximates Hessian's inverse only on the basis of knowledge of gradient changes of criterion J and decision variables $\boldsymbol{x}$ in two consecutive iterations. In each iteration to improve matrix $\boldsymbol{B}$ the following formula is used (Papalambros and Wilde [7]):

$$\boldsymbol{B}_{k+1}^{(BFGS)} = \boldsymbol{B}_k + \left[1 + \frac{\partial \boldsymbol{v}^T \boldsymbol{B} \partial \boldsymbol{v}}{\partial \boldsymbol{x}^T \partial \boldsymbol{v}}\right]_k \cdot \left[\frac{\partial \boldsymbol{x} \partial \boldsymbol{x}^T}{\partial \boldsymbol{x}^T \partial \boldsymbol{v}}\right]_k - \left[\frac{\partial \boldsymbol{x} \partial \boldsymbol{v}^T \boldsymbol{B} + \boldsymbol{B} \partial \boldsymbol{x} \partial \boldsymbol{v}^T}{\partial \boldsymbol{x}^T \partial \boldsymbol{v}}\right]_k \tag{23}$$

where:

$$\partial \boldsymbol{x} = \boldsymbol{x}_{k+1} - \boldsymbol{x}_k \tag{24}$$

$$\partial \boldsymbol{v} = \nabla J(\boldsymbol{x}_{k+1}) - \nabla J(\boldsymbol{x}_k) \tag{25}$$

It is frequently assumed that $\boldsymbol{B}_0 = \boldsymbol{I}$, where $\boldsymbol{I}$ denotes an identity matrix.

Usually estimators of nonlinear models are biased and up till now there is no method to calculate the error of bias. It is only possible to determine the lower bound of the variance $\boldsymbol{D}^2(\hat{\boldsymbol{x}})$ of the unbiased estimates. For this purpose the Cramer-Rao information inequality is used [5]. From this inequality and assuming normal distribution N($\mathbf{0}$, $\boldsymbol{R}$) of the vector of random errors of observation we obtain:

$$\boldsymbol{D}^2(\hat{\boldsymbol{x}}) \geq [\boldsymbol{H}^{(1)T}(\hat{\boldsymbol{x}}) \cdot \boldsymbol{H}^{(1)}(\hat{\boldsymbol{x}})]^{-1} \cdot \boldsymbol{s}^2 \tag{26}$$

where: $\hat{\boldsymbol{x}}$ – vector of estimates of identified parameters, $\boldsymbol{H}^{(1)}(\hat{\boldsymbol{x}})$ – Jacobian matrix of the vector function $\boldsymbol{q}(\boldsymbol{x})$,

$$\boldsymbol{H}^{(1)}(\hat{\boldsymbol{x}}) = [\boldsymbol{f}_1(\hat{\boldsymbol{x}}), \boldsymbol{f}_2(\hat{x}), ..., \boldsymbol{f}_r(\hat{\boldsymbol{x}})] \tag{27}$$

where:

$$\boldsymbol{f}_i(\boldsymbol{x}) = \left[\frac{\partial \ddot{q}_1(\boldsymbol{x})}{\partial x_i}, \frac{\partial \ddot{q}_2(\boldsymbol{x})}{\partial x_i}, ..., \frac{\partial \ddot{q}_n(\boldsymbol{x})}{\partial x_i} \right]^T \tag{28}$$

Variance $\boldsymbol{s}^2$ is calculated according to the formula:

$$s^2 = \frac{1}{N - r - 1} \sum_{i=1}^{N} \sum_{j=1}^{n} (\ddot{z}_{ij} - \ddot{q}_{ij}(\hat{\boldsymbol{x}}))^2 \tag{29}$$

where: N – total number of points from the characteristics (acceleration time response series) on the basis of which identification of the model is carried out, r – number of identified parameters, n – number of characteristics on the basis of which the identification procedure is carried out.

4. Numerical example

4.1. Model description

The presented algorithm has been implemented in Matlab as a software package ***IDENTNL-INCI***. The software was tested using simulated data for nonlinear model of the isolated knee–column system of the universal milling machine, described in details in paper [4].

This model (Fig. 1) consists of two rigid finite elements (RFEs) and nine spring-damping elements (SDEs). The RFEs model the two main bodies of the milling machine's supporting system, i.e. the knee and the column. In a real machine these elements are connected by a slideway system that enables vertical motion of the knee.

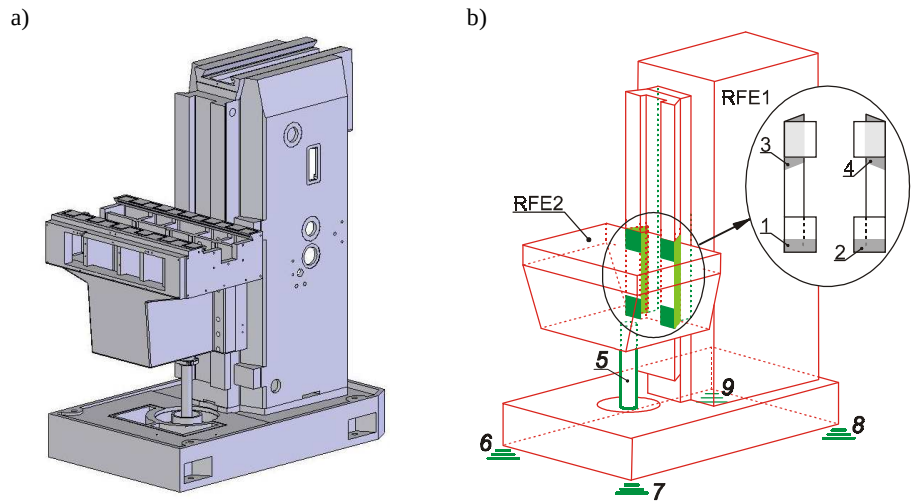

Fig. 1. Knee–column system of tested milling machine: a) general view, b) RFE model showing active spring–damping elements [4]

On the basis of the results of the earlier experiments carried out by Gutowski [8], and Gutowski and Berczyński [9] which revealed considerable nonlinearity in knee–column joint it was assumed that SDEs no 1–4 modeling particular working slideways of this joint have in normal directions nonlinear characteristics. In tangential directions, however, it was assumed that spring characteristics of this joint are linear.

Vibration isolation pads (SDEs no 6–9) and the ball screw of the knee (SDE no 5) feed drive system are modeled as linear elements.

Identified are only parameters of four SDEs modeling knee–column joint. Each SDE in this joint is represented by four parameters. Two of them characterize nonlinear contact stiffness (a_{ni}) in normal direction and linear contact

stiffness (a_{ti}) in tangential direction, and the other two characterize linear damping (h_{ni}) and (h_{ti}) of the contact in these directions. In described example of identification on the basis of incomplete information of the object only the acceleration time response characteristics of the knee defined in the system of principal central axes of inertia of this body were used.

The data was simulated using a model with known parameters (obtained from the identification of the linear model) that characterize spring-damping parameters of the slideway joint. The harmonic force $P(t) = P_0 \sin\omega t$ was used for excitation purposes. The amplitude of excitation was P_0 = 100 daN. The excitation force acted at the same time on the knee and the column. It was generated by the hydraulic exciter fitted to the knee. It was assumed, in the computation, that the exponent m in equation (6) in the case of nonlinear SDEs is equal to 2 (Levina and Reshetov [10]).

According to the described algorithm, for the incomplete information variant of identification, a two steps approach was implemented. First, up to the elaborated plan of experiment, an optimum starting point was searched in the assumed space of parameters variability. Then, starting from this point, the minimum of the objective function was calculated using iterative variable metric optimization routines.

4.2. Results of the identification

In Figure 2 the acceleration response signals of the knee determined in directions of the three principal central axes of inertia of this body are shown. On the basis of these signals the identification was carried out in described case of incomplete information of the object.

The results are compared in Tables 1 and 2. These tables show the true values of the identified parameters and their estimates, obtained in the conducted computations. It is seen that the values of estimates determined on the basis of incomplete information of the object do not differ significantly from the actual values of parameters. Mostly, the relative error between estimated and actual value of parameter does not exceed 2.5%. The only significant error (11.4%) is observed for the 15th decision variable (tangential damping h_{t3}).

We don't discuss in-depth in this paper the efficiency of the elaborated procedures because only several examples were calculated. Additionally, in this paper we have shown results only of one of them. Some information about this efficiency is given in the form of total number of iterations and time necessary for performing calculations in particular iteration steps. But it is obvious that the time of calculations depends not only on the efficiency of elaborated algorithm but also on the efficiency of processor installed in a used computer.

For presented in the paper example the calculations were carried out according to the LP_t plan with 100 points, each of them lasted 30 seconds, and

next for the best point they were continued in the iterative procedure which needed 580 iteration steps, each step lasting 40 seconds.

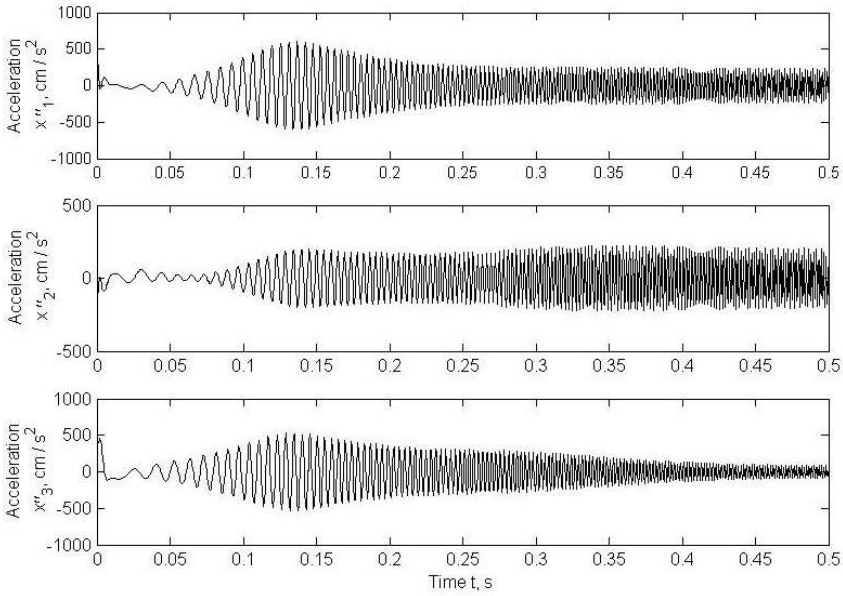

Fig. 2. Acceleration time response series of the knee in directions of principal central axes of inertia, used in incomplete information identification

Table 1. Results of the identification of the nonlinear normal and linear tangential stiffnesses a_n and a_t of the SDEs no 1–4 for the case of incomplete information of the object

No of variable	**Kind of parameter**	**Area of searching daN μm^{-2}**		**Value of parameter daN μm^{-2}**		**Standard error** *
		min.	**max.**	**actual**	**identified**	
1	a_{n1}	190	570	380	385.3	33.716
2	a_{n2}	185	555	370	377.5	15.337
3	a_{n3}	170	510	340	334.8	21.534
4	a_{n4}	330	990	660	666.9	10.365
No of variable	**Kind of parameter**	**Area of searching daN μm^{-1}**		**Value of parameter daN μm^{-1}**		**Standard error** *
		min.	**max.**	**actual**	**identified**	
5	a_{t1}	7.5	22.5	15	14.99	0.02204
6	a_{t2}	7.5	22.5	15	15.46	0.02847
7	a_{t3}	6.5	19.5	13	13.02	0.00717
8	a_{t4}	13.5	40.5	27	26.44	0.01818

*– square root of the lower bound of the estimate variance

Table 2. Results of the identification of the linear normal and tangential dampings h_n and h_t of the SDEs no 1–4 for the case of incomplete information of the object

No of variable	Kind of parameter	Area of searching daN μm^{-1}		Value of parameter daN μm^{-1}		Standard error*
		min.	max.	actual	identified	
9	h_{n1}	1.8	5.4	3.6	3.572	0.00878
10	h_{n2}	1.75	5.25	3.5	3.487	0.01299
11	h_{n3}	1.35	4.95	2.7	2.760	0.01457
12	h_{n4}	1.05	3.15	2.1	2.021	0.00645
13	h_{t1}	2.75	8.25	5.5	5.596	0.01278
14	h_{t2}	2.75	8.25	5.5	5.291	0.01666
15	h_{t3}	0.105	0.315	0.21	0.234	0.00471
16	h_{t4}	3.85	11.55	7.7	7.764	0.01133

*– square root of the lower bound of the estimate variance

The convergence of iteration algorithm was tested by means of carrying out calculations using different starting points for model characteristics not biased by errors – generated for the detuned model. Every time, after finishing iteration procedures, the determined values of sought for estimates of model parameters were within the tolerance assumed to stop the carried out calculations.

5. Conclusions

The performed numerical tests showed a very good convergence of the elaborated computational procedures when complete information about the object could not be obtained. This is especially important in the identification of real objects which do not make it possible to measure a full set of signals (displacements, velocities and accelerations) of their particular bodies and therefore it is necessary to carry out identification on the basis of limited number of such characteristics. It should also be emphasized that a good con-vergence of the elaborated procedures was achieved for a large number (n = 16) of decision variables.

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